Math 340
Sample First Midterm

[**12 points**] **1(a)** Solve the following linear programming problem, using the methods studied in class:

$$\begin{array}{lrrrl} \text{maximize} & x_1 & +3x_2 & -x_3 & \\ \text{subject to} & & 2x_2 & -2x_3 & \leq -2 \\ & -x_1 & & -2x_3 & \leq -1 \\ & x_1 & +x_2 & +x_3 & \leq 2 \\ & x_1, x_2, x_3 \geq 0 & & & \end{array}$$

[**4 points**] **(b)** What does the result of the **first** pivot allow you to conclude about the value of the objective in an optimal solution?

[**4 points**] **(c)** How could you change one number in this problem so that the result of the first pivot would be a degenerate tableau?

2. Consider the problem

$$\begin{array}{lrrrrl} \text{maximize} & x_1 & +9x_2 & +x_3 & +2x_4 & \\ \text{subject to} & x_1 & +8x_2 & +x_3 & +2x_4 & \leq 9 \\ & 3x_1 & +12x_2 & -3x_3 & -3x_4 & \leq 18 \\ & & x_2 & +x_3 & +2x_4 & \leq 1 \\ & x_1, x_2, x_3, x_4 \geq 0 & & & & \end{array}$$

An optimal solution is found with basis x_1, s_2, x_2 (where s_2 is the slack variable corresponding to the second constraint), and

$$\mathbf{B}^{-1} = \begin{array}{c} x_1 \\ s_2 \\ x_2 \end{array} \begin{pmatrix} 1 & 0 & -8 \\ -3 & 1 & 12 \\ 0 & 0 & 1 \end{pmatrix}$$

[**6 points**] **(a)** For what values of the parameter t would this basis give the optimal solution, if we changed the right hand side of the second constraint to $t + 18$?

[**7 points**] **(b)** Suppose (with the original $\mathbf{b}$), the objective was changed to: maximize $x_1 + 8x_2 + x_3 + 3x_4$. Would the optimal solution change? If a pivot is necessary, which variables would enter and leave the basis?

[**7 points**] **3.** Suppose a linear programming problem P is unbounded. How would that fact be detected in the Simplex Method? Explain carefully why we know that P is unbounded when this occurs.