Marks

[8] **1.** Consider the following tableau for a standard primal linear programming problem.

z	x_1	x_2	x_3	s_1	s_2	rhs		
1	0	p	0	5	3	14	=	z
0	1	q	0	1	0	1	=	x_1
0	0	r	1	2	1	5	=	x_3

For each of the following questions (separately), give an example of values of p, q and r to make the statement true.

(a) The basic solution for this tableau is the unique optimal solution.

(b) The basic solution for this tableau is optimal but is not the only optimal solution, and there is another optimal solution with $x_2 = 2$.

(c) The basic solution for this tableau is not optimal, and the basic solution for the next tableau (using the usual Simplex Method) will have $x_2 = 2$, $x_3 = 1$ and $z = 20$.

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[8] **2.** Solve the following linear programming problem, using the Simplex Method.

$$
\begin{array}{lrrrl}
\text{maximize } z = & -3x_1 & +3x_2 & +4x_3 & \\
\text{subject to} & x_1 & +x_2 & +x_3 & \leq 30 \\
 & -2x_1 & +x_2 & +2x_3 & \geq 12 \\
 & x_1, x_2, x_3 \geq 0 & & &
\end{array}
$$

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[8] **3.** Perform **one** iteration of the Dual Simplex Method on the following linear programming problem. What does the result tell you about the optimal value, if any, of the objective?

$$
\begin{array}{llrrrl}
\text{maximize } z = & -3x_1 & -4x_2 & -x_3 & \\
\text{subject to} & x_1 & +x_2 & & \leq 3 \\
& -x_1 & -2x_2 & +x_3 & \leq -1 \\
& -2x_1 & & -x_3 & \leq -2 \\
& x_1, x_2, x_3 \geq 0 & & &
\end{array}
$$

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[10] **4.** Consider the linear programming problem

$$\begin{array}{lrrrl}
\text{maximize} & 2x_1 & +x_2 & +4x_3 & \\
\text{subject to} & x_1 & +x_2 & +x_3 & \leq 13 \\
 & 2x_1 & +3x_2 & +2x_3 & \leq 22 \\
 & x_1 & +2x_2 & +x_3 & = 11 \\
 & \multicolumn{4}{l}{x_1, x_2, x_3 \geq 0}
\end{array}$$

Your friend claims that the optimal solution has $x_1 = x_2 = 0$, $x_3 = 11$. Confirm this using Complementary Slackness, and find all optimal solutions of the dual problem. DO NOT USE THE SIMPLEX METHOD, WHETHER PRIMAL OR DUAL, IN THIS QUESTION.

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5. A linear programming problem in standard form has the optimal tableau

z	x_1	x_2	x_3	s_1	s_2	rhs		
1	0	3	0	5	3	14	$=$	z
0	1	-1	0	1	0	1	$=$	x_1
0	0	2	1	2	1	5	$=$	x_3

Note that each of the following parts is independent of the others, i.e. when doing one part you do not consider changes made in the other parts.

[7] (a) What would be the effect on the basic variables and the objective of adding a small number ϵ (positive or negative) to the right side of the first constraint of the original problem? For what interval of values of ϵ would this be true? If ϵ was slightly outside this interval, what variables would enter and leave the basis?

[5] (b) Suppose a new variable x_4 is introduced, with (in the original problem) coefficient p in the first constraint and $-p$ in the second, and coefficient 3 in the objective. What must be true about p if an optimal solution of the new problem has $x_4 > 0$?

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[6] (c) Suppose we introduce a new constraint $x_1 + x_3 \leq 6$. What new row and column would be this introduce into the tableau? What would happen to the optimal solution?

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[13] **6.** Consider the linear programming problem

$$\begin{array}{lrrrrrcl} \text{maximize} & x_1 & +2x_2 & +4x_3 & +3x_4 & +2x_5 & & \\ \text{subject to} & x_1 & +x_2 & -2x_3 & +x_4 & +x_5 & \le & 1 \\ & x_1 & +2x_2 & +3x_3 & -3x_4 & +3x_5 & \le & 12 \\ & x_1 & & -x_3 & & +x_5 & \le & 3 \\ & \multicolumn{7}{l}{x_1, x_2, x_3, x_4, x_5 \ge 0} \end{array}$$

In the process of solving it with the Revised Simplex Method, suppose we reach the basis x_1, s_2, x_3 with

$$B^{-1} = \begin{pmatrix} -1 & 0 & 2 \\ 4 & 1 & -5 \\ -1 & 0 & 1 \end{pmatrix}$$

Perform the next pivot, obtaining the new basis, B^{-1} and $\boldsymbol{\beta}$.

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7. A small firm produces four coffee blends (C_1, C_2, C_3 and C_4), each of which consists of a mixture of Brazilian, Colombian and Peruvian coffees. The table below gives the proportions of the different coffees making up each blend, the profit per kilogram of each blend sold, and the maximum amount of each coffee available for this week's production. The firm would like to choose amounts of each blend to produce so as to maximize its profits.

		Coffee			
		Brazilian	Colombian	Peruvian	Profit ($) per kg
Blend	C_1	20%	40%	40%	0.8
	C_2	40%	50%	10%	0.6
	C_3	30%	30%	40%	0.4
	C_4	70%	20%	10%	0.5
Availability(kg)		8000	6400	6000	

The following file is submitted to LINDO:

```
max 0.8 x1 + 0.6 x2 + 0.4 x3 + 0.5 x4
st
p1) 0.2 x1 + 0.4 x2 + 0.3 x3 + 0.7 x4 <= 8000
p2) 0.4 x1 + 0.5 x2 + 0.3 x3 + 0.2 x4 <= 6400
p3) 0.4 x1 + 0.1 x2 + 0.4 x3 + 0.1 x4 <= 6000
end
```

The results are as follows:

```
.LP OPTIMUM FOUND AT STEP      3
.
.        OBJECTIVE FUNCTION VALUE
.
.        1)      13600.00
.
.  VARIABLE        VALUE          REDUCED COST
.        X1     12000.000000          0.000000
.        X2         0.000000          0.425000
.        X3         0.000000          0.225000
.        X4      8000.000000          0.000000

.       ROW   SLACK OR SURPLUS     DUAL PRICES
.       P1)         0.000000          0.166667
.       P2)         0.000000          1.916667
.       P3)       400.000000          0.000000
.
. NO. ITERATIONS=       3

. RANGES IN WHICH THE BASIS IS UNCHANGED:
.
.                            OBJ COEFFICIENT RANGES
. VARIABLE         CURRENT         ALLOWABLE        ALLOWABLE
.                   COEF           INCREASE         DECREASE
.       X1        0.800000         0.200000         0.360000
.       X2        0.600000         0.425000         INFINITY
```

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```
.        X3         0.400000          0.225000          INFINITY
.        X4         0.500000          2.300000          0.100000
.
.                             RIGHTHAND SIDE RANGES
.       ROW          CURRENT         ALLOWABLE         ALLOWABLE
.                       RHS           INCREASE          DECREASE
.        P1      8000.000000      14400.000000       2400.000000
.        P2      6400.000000        369.230743       4114.285645
.        P3      6000.000000          INFINITY        400.000000
```

[4] (a) Briefly explain the practical significance to the firm of each entry in the `RIGHTHAND SIDE RANGES` for P2.

Answer the following questions using the computer output - do NOT try to solve the linear programming problem yourself.

[5] (b) Suppose, in addition to the 6400 kg of Colombian coffee available from the usual supplier, there is an opportunity to buy some Colombian coffee from another supplier, at a somewhat higher price. Under what conditions should the firm buy from that source? What can you say about how much should be bought?

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[5] (c) The firm is considering a change in the relative proportions of Brazilian and Colombian coffees in blend C_3. All other data, including the profit per kilogram, would remain the same. Might this be a good idea? If so, which of the two should be increased, and by how much?

[5] (d) An across-the-board increase in the prices of the three blends is being considered. This would raise the profit per kilogram of all four blends by the same positive amount p. For what values of p can we be sure that the optimal solution will not change?

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8. Consider the nonlinear programming problem

$$\begin{aligned} \text{maximize } & x_1 + 2x_2 \\ \text{subject to } & x_1 + x_2 \leq 1 \\ & x_1^2 + x_2^2 \leq 5 \end{aligned}$$

[7] (a) State the Karush-Kuhn-Tucker conditions for this problem.

[5] (b) Find a solution to the Karush-Kuhn-Tucker conditions. *Hint:* λ_1 and λ_2 are both nonzero.

[4] (c) How can you be sure that the solution found in (b) is the global maximum?

The End

Be sure that this examination has 12 pages including this cover

The University of British Columbia

Sessional Examinations - April 2003

Mathematics 340

Linear Programming

Dr. Israel

Closed book examination | Time: 2 $\frac{1}{2}$ hours

First Name ____________________ **Last Name** ____________________

Student Number____________________ **Signature** ____________________

Special Instructions:

Allowed aids: A non-graphing, non-programmable calculator, and one sheet (two-sided) of notes.
Write your name at the top of each page. Write your answers in the space provided. Show your work.

Rules governing examinations

1. Each candidate should be prepared to produce his or her library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

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3. Smoking is not permitted during examinations.

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3		8
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7		19
8		16
Total		100