The Simplex Method, Phase II

Assumptions:
All variables (including slack variables, but not the objective z) are required to be ≥ 0.
We have a tableau where the basic solution is feasible. This will be maintained throughout our pivoting.

Pivot Selection

1. Choose a nonbasic variable with a negative coefficient in the objective row. This will be the **entering variable**.
   a) If there are none, stop: the tableau is optimal.
   b) If there are several possible entering variables, we usually choose one with the most negative coefficient (this is the **most negative coefficient rule**); but other choices are possible.
   c) In case of a tie for most negative, we'll choose the one farthest left.
2. For each positive entry in the entering variable's column, calculate the ratio

$$\frac{\text{Right-hand side entry}}{\text{Coefficient of entering variable}}$$

   a) Choose the smallest ratio. The basic variable for the row where this occurs is the **leaving variable**.
   b) If there are no ratios to calculate, because all entries in the entering variable's column are negative or zero, stop: the problem is unbounded.
   c) In case of a tie for smallest ratio, we'll choose the one highest up in the tableau.

Pivoting

The pivot row and pivot column are the row and column labelled by the leaving and entering variables respectively. The pivot entry is the entry in the tableau in the pivot row and pivot column.

3. Divide the pivot row by the pivot entry.
4. Add the appropriate multiple of the pivot row to each other row to make the entry in the pivot column for that row 0.
5. Relabel the pivot row by the entering variable.
6. Return to step 1.

Example

$$
\begin{array}{ll}
\text{maximize} & z = 3\,x_1 + 2\,x_2 \\
\text{subject to} & \quad 2\,x_1 - \;x_2 \le 1 \\
& -3\,x_1 + 4\,x_2 \le 13 \\
& \quad x_1 + \;x_2 \le 5 \\
& \quad x_1, x_2 \ge 0
\end{array}
$$

Initial tableau:

z	x_1	x_2	s_1	s_2	s_3	rhs		
1	-3	-2	0	0	0	0	$=$	z
0	2	-1	1	0	0	1	$=$	s_1
0	-3	4	0	1	0	13	$=$	s_2
0	1	1	0	0	1	5	$=$	s_3

x_1 enters, ratios $1/2$ for s_1 and $5/1$ for s_3. s_1 leaves.

z	x_1	x_2	s_1	s_2	s_3	rhs		
1	0	$-7/2$	$3/2$	0	0	$3/2$	$=$	z
0	1	$-1/2$	$1/2$	0	0	$1/2$	$=$	x_1
0	0	$5/2$	$3/2$	1	0	$29/2$	$=$	s_2
0	0	$3/2$	$-1/2$	0	1	$9/2$	$=$	s_3

x_2 enters, s_3 leaves.

z	x_1	x_2	s_1	s_2	s_3	rhs		
1	0	0	$1/3$	0	$7/3$	12	$=$	z
0	1	0	$1/3$	0	$1/3$	2	$=$	x_1
0	0	0	$7/3$	1	$-5/3$	7	$=$	s_2
0	0	1	$-1/3$	0	$2/3$	3	$=$	x_2

This is optimal!

An unbounded example

$$
\begin{array}{ll}
\text{maximize} & z = 3\,x_1 + 2\,x_2 \\
\text{subject to} & \;\;x_1 - \;x_2 \;\;\le 1 \\
& -x_1 + \;x_2 \le 2 \\
& 2\,x_1 - 3\,x_2 \le 2 \\
& x_1, x_2 \ge 0
\end{array}
$$

z	x_1	x_2	s_1	s_2	s_3	rhs		
1	−3	−2	0	0	0	0	=	z
0	1	−1	1	0	0	1	=	s_1
0	−1	1	0	1	0	2	=	s_2
0	2	−3	0	0	1	2	=	s_3

x_1 enters. Tie for leaving variable: we choose s_1.

z	x_1	x_2	s_1	s_2	s_3	rhs		
1	0	−5	3	0	0	3	=	z
0	1	−1	1	0	0	1	=	x_1
0	0	0	1	1	0	3	=	s_2
0	0	−1	−2	0	1	0	=	s_3

x_3 enters; no ratios to calculate, so problem is unbounded.