Parametric programming for an objective coefficient

Consider the following problem:

$$\begin{array}{lll} \text{maximize} & z = 20x_1 + 30x_2 + 14x_3 & \\ \text{subject to} & 3/2\,x_1 + x_2 + x_3 & \le 100 \\ & x_1 + 2x_2 + x_3 & \le 100 \\ & x_1 + x_2 + 3x_3 & \le 120 \\ & x_1, x_2, x_3 \ge 0 & \end{array}$$

Here is the optimal tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	0	7/2	5	25/2	0	1750	=	z
0	1	0	1/2	1	−1/2	0	50	=	x_1
0	0	1	1/4	−1/2	3/4	0	25	=	x_2
0	0	0	9/4	−1/2	−1/4	1	45	=	s_3

Suppose we change the coefficient of x_1 in the objective: instead of 20 it will be $20 + \varepsilon$.

The new z is the old $z + \varepsilon x_1$. So the new z row is obtained by adding ε times the x_1 row (except for the entry in the x_1 column) to the z row:

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	0	$7/2 + \varepsilon/2$	$5 + \varepsilon$	$25/2 - \varepsilon/2$	0	$1750 + 50\varepsilon$	=	z
0	1	0	1/2	1	−1/2	0	50	=	x_1
0	0	1	1/4	−1/2	3/4	0	25	=	x_2
0	0	0	9/4	−1/2	−1/4	1	45	=	s_3

This tableau is optimal as long as the entries in the z row for nonbasic variables remain ≥ 0:

$$\begin{array}{lll} 7/2 + \varepsilon/2 \ge 0 & \text{i.e.} & \varepsilon \ge -7 \\ 5 + \varepsilon \ge 0 & \text{i.e.} & \varepsilon \ge -5 \\ 25/2 - \varepsilon/2 \ge 0 & \text{i.e.} & \varepsilon \le 25 \end{array}$$

Thus the interval is $-5 \le \varepsilon \le 25$. If $\varepsilon > 25$, s_2 would enter the basis and x_2 would leave, resulting in the following tableau:

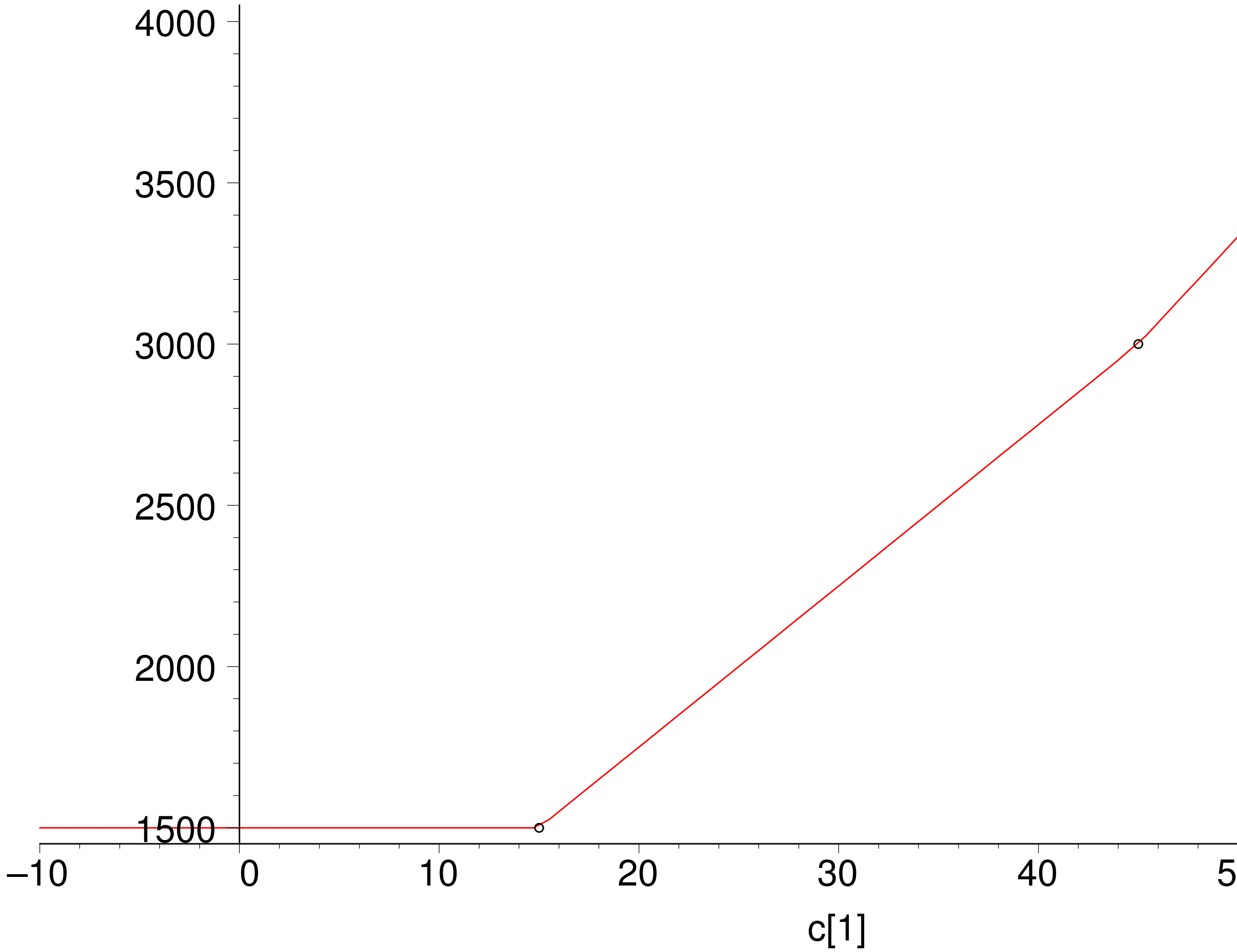
z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	$-50/3$ $+2\varepsilon/3$	$-2/3$ $+2\varepsilon/3$	$40/3$ $+2\varepsilon/3$	0	0	$4000/3$ $+200\varepsilon/3$	=	z
0	1	$2/3$	$2/3$	$2/3$	0	0	$200/3$	=	x_1
0	0	$4/3$	$1/3$	$-2/3$	1	0	$100/3$	=	s_2
0	0	$1/3$	$7/3$	$-2/3$	0	1	$160/3$	=	s_3

This is optimal if $\varepsilon \geq 25$.

Returning to the previous tableau, if $\varepsilon < -5$, s_1 would enter and x_1 would leave, resulting in the tableau:

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	$-5-\varepsilon$	0	1	0	15	0	1500	=	z
0	1	0	$1/2$	1	$-1/2$	0	50	=	s_1
0	$1/2$	1	$1/2$	0	$1/2$	0	50	=	x_2
0	$1/2$	0	$5/2$	0	$-1/2$	1	70	=	s_3

If we graph the optimal value of the objective as a function of the objective coefficient c_1 of x_1, it looks like this:


4000
3500
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