Math 340: Solutions to First Midterm

Oct. 18, 2006

[15] **1**. *Solve the following linear programming problem, using the methods studied in class:*

maximize

$-2x_1 + x_2 + x_3$

subject to

$-x_1 - x_2 \le -1$

$2x_1 + x_2 - 3x_3 \le 4$

$x_1, x_2, x_3 \ge 0$

Since the initial basic solution would not be feasible, we need a Phase I. We introduce an artificial variable a_0 so the first constraint becomes $-x_1 - x_2 + s_1 - a_0 = -1$, and the temporary objective is $w = -a_0$. The first tableau is

w	z	x_1	x_2	x_3	s_1	s_2	a_0	rhs		
1	0	0	0	0	0	0	1	0	=	w
0	1	2	−1	−1	0	0	0	0	=	z
0	0	−1	−1	0	1	0	−1	−1	=	s_1
0	0	2	1	−3	0	1	0	4	=	s_2

In the first pivot, a_0 enters and s_1 leaves, so that the basic solution will be feasible for the relaxed problem.

w	z	x_1	x_2	x_3	s_1	s_2	a_0	rhs		
1	0	−1	−1	0	1	0	0	−1	=	w
0	1	2	−1	−1	0	0	0	0	=	z
0	0	1	1	0	−1	0	1	1	=	a_0
0	0	2	1	−3	0	1	0	4	=	s_2

Now x_1 enters and a_0 leaves.

w	z	x_1	x_2	x_3	s_1	s_2	a_0	rhs		
1	0	0	0	0	0	0	1	0	=	w
0	1	0	−3	−1	2	0	−2	−2	=	z
0	0	1	1	0	−1	0	1	1	=	x_1
0	0	0	−1	−3	2	1	−2	2	=	s_2

Since $w = 0$, we have a successful conclusion of Phase I: the basic solution is feasible for the original problem. We delete the w row and column, and continue with Phase II where z is the objective. Now x_2 enters and x_1 leaves.

z	x_1	x_2	x_3	s_1	s_2	a_0	rhs		
1	3	0	−1	−1	0	1	1	=	z
0	1	1	0	−1	0	1	1	=	x_2
0	1	0	−3	1	1	−1	3	=	s_2

Here x_3 enters, but there are no ratios to calculate, so we conclude the problem is unbounded.

[15] **2**. *Consider the problem*
maximize
$2x_1+14x_2+8x_3$
subject to
$2x_1 \ +2x_2 \ +x_3 \leq 9$
$-3x_1 \ \ +x_2 \ +x_3 \leq 7$
x_1 URS, $x_2, x_3 \geq 0$

(a) *Find the tableau for the basis x_2, x_3, using the fact that* $B^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$. *Do* **not** *use pivoting.*

$$B^{-1}N = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} \overset{x_1}{2} & \overset{s_1}{1} & \overset{s_2}{0} \\ -3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \overset{x_1}{5} & \overset{s_1}{1} & \overset{s_2}{-1} \\ -8 & -1 & 2 \end{pmatrix},$$ which goes in the main body of the tableau in the x_1, s_1 and s_2 columns.

$\boldsymbol{\beta} = B^{-1}\begin{pmatrix} 9 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ which goes in the main part of the rhs column.

$\mathbf{y}^T = (14 \quad 8)\, B^{-1} = (6 \quad 2)$ and $\boldsymbol{\eta}_{NBV} = \mathbf{y}^T N - \begin{pmatrix} \overset{x_1}{2} & \overset{s_1}{0} & \overset{s_2}{0} \end{pmatrix} = \begin{pmatrix} \overset{x_1}{4} & \overset{s_1}{6} & \overset{s_2}{2} \end{pmatrix}$ which goes in the objective row.

$z^* = \mathbf{c}_{BV}^T \boldsymbol{\beta} = 68$, which goes in the rhs column of the objective row.

Thus the tableau is

z	x_1^U	x_2	x_3	s_1	s_2	rhs		
1	4	0	0	6	2	68	=	z
0	5	1	0	1	-1	2	=	x_2
0	-8	0	1	-1	2	5	=	x_3

(b) *Is the basic solution for this tableau optimal? If not, what variables should enter and leave the basis in a pivot starting from this tableau?*

It is not optimal, because the entry for the URS variable x_1 in the objective row is nonzero. x_1 should enter (decreasing), and x_3 should leave.

[10] **3**. *Suppose we use the Simplex Method with the "most negative entry" rule. If no degenerate pivots occur, this will eventually stop. Why? Why might this not be true if there are degenerate pivots?*

If there are no degenerate pivots, the objective increases at each pivot, and we never see the same tableau more than once. Since there are only a finite number of possible tableaus, the process must eventually stop. If there are degenerate pivots, cycling can occur, which would mean the Simplex Method goes on forever.