Assignment 8

Due Wednesday, Nov. 15

10.1.1

E.1. A manufacturer produces coats, jackets and sweaters. The numbers of each of these to produce in a week, x_1 to x_3, are found by solving the linear programming problem

$$\begin{array}{lll} \text{maximize} & z = 6x_1 + 4x_2 + 5x_3 & \\ \text{subject to} & 3x_1 + 2x_2 + x_3 & \le 80 \\ & 2x_1 + x_2 + 2x_3 & \le 70 \\ & x_3 & \le 30 \\ & x_1, x_2, x_3 \ge 0 & \end{array}$$

where the right sides of the three constraints are the amounts of time available each week on three machines M1, M2, M3. The optimal solution is found with basis x_2, x_3, s_3, and has

$$B^{-1} = \begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 1/3 & -2/3 & 1 \end{pmatrix}$$

(a). Suppose the amount of time available on M2 is p instead of 70. For what values of p would the basis above still give an optimal solution? How would the values of the variables and the objective depend on p in this interval? Find all the intervals for p in which different bases give optimal solutions. Sketch the graph of the optimal objective value as a function of p.

(b). Suppose we add (to the original problem) the additional requirement that no more than 40 items in total can be produced in a week. What would be the new optimal solution?

E.2. Solve, using the Revised Simplex Method:

$$\begin{array}{ll} \text{maximize} & z = 13x_1 + 10x_2 + 12x_3 + 17x_4 \\ \text{subject to} & 3x_1 + x_2 + 2x_3 + 2x_4 \le 8 \\ & 4x_1 + 3x_2 + 4x_3 + 5x_4 \le 21 \\ & \text{all } x_j \ge 0 \end{array}$$

E.3. Professor Bumble was trying to solve the problem

$$\begin{array}{lll} \text{maximize} & z = x_1 + 2x_2 + 6x_3 & \\ \text{subject to} & -x_1 + 5\,x_2 + 60\,x_3 + x_4 & \le 50 \\ & -x_1 - 2\,x_2 - 4\,x_3 + x_4 & \le -21 \\ & 3\,x_1 + 5\,x_2 + 3\,x_3 + 2\,x_4 & \le 53 \\ & \text{all } x_j \ge 0 & \end{array}$$

He had arrived at the basis x_1, x_2, s_1, where

$$B^{-1} = \begin{pmatrix} 0 & 5 & 2 \\ 0 & -3 & -1 \\ 1 & 20 & 7 \end{pmatrix}$$

and then was called away to an emergency meeting of the faculty. Finish the Professor's work, using the Revised Simplex Method, starting with his basis.