Assignment 7

Due Monday, Nov. 6

6.8.2, 6.8.7, 6.8.9, 6.11.1, 6.11.3

E.1. Use Complementary Slackness to check whether $x_1 = 3$, $x_2 = -1$, $x_3 = 0$, $x_4 = 2$ is an optimal solution of the problem

$$\begin{aligned}
\text{maximize} \quad & z = 6x_1 + x_2 - x_3 - x_4 \\
\text{subject to} \quad & x_1 + 2x_2 + x_3 + x_4 \leq 5 \\
& 3x_1 + x_2 - x_3 \leq 8 \\
& x_2 + x_3 + x_4 = 1 \\
& x_1 \text{ and } x_2 \text{ URS, } x_3 \text{ and } x_4 \geq 0
\end{aligned}$$

E.2(a). Professor Bumble wants to assign his class a linear programming problem P of the form

$$\begin{aligned}
\text{maximize} \quad & z = c_1x_1 + 2x_2 + x_3 \\
\text{subject to} \quad & x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\
& x_1 - x_3 \leq 4 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

He wants $x_1 = 6$, $x_2 = 0$, $x_3 = 4$ to be an optimal solution of P, and $y_1 = -1$, $y_2 = 0$ to be an optimal solution of the dual D. What should the constants c_1, a_{12}, a_{13} and b_1 be?

Hint: Use Complementary Slackness.

(b). After following your advice, the Professor is horrified to find that nobody in the class got the answer he wanted. What went wrong?

Hint: Is the Professor's solution a basic solution?

E.3. 3000 years from now, an archaeologist discovers a sheet of paper in the ruins of the Math Annex. On it is a linear programming problem, but part of it is illegible:

$$\begin{aligned}
\text{maximize} \quad & z = (\text{smudge}) - 34x_2 + (\text{ smudge}) \\
\text{subject to} \quad & 6x_1 - 4x_2 + 3x_3 \leq 4 \\
& 9x_1 - 6x_2 \leq 6 \\
& -3x_1 + 2x_2 + 7x_3 \leq -1 \\
& x_1, x_2 \text{ URS, } x_3 \geq 0
\end{aligned}$$

The only other thing that can be read is a note that the dual problem has an optimal solution with $y_2 = 3$.

What is the optimal value of the objective? What else can you determine about the problem?

Hint: Use Complementary Slackness, and the equations of both the primal and dual problems.