Math 340: Answers to Assignment 6

6.5.3. We write the third constraint as $-2x_2 - x_3 \le -6$. Since x_2 and x_3 are URS, the second and third constraints of the dual are equalities. Since the fourth constraint is an equality, y_4 is URS. Thus the dual is

$$\begin{array}{lllllll} \min & 5y_1 & +7y_2 & -6y_3 & +4y_4 & & \\ \text{s.t.} & y_1 & +2y_2 & & +y_4 & \ge & 4 \\ & y_1 & +y_2 & -2y_3 & & = & -1 \\ & & & -y_3 & +y_4 & = & 2 \\ & y_1, y_2, y_3 \ge 0,\ y_4 URS & & & & & \end{array}$$

6.7.1(a). The dual is

$$\begin{array}{llllll} \min & 100y_1 & +80y_2 & +40y_3 & & \\ \text{s.t.} & 2y_1 & +y_2 & +y_3 & \ge & 3 \\ & y_1 & +y_2 & & \ge & 2 \\ & y_1, y_2, y_3 \ge 0 & & & & \end{array}$$

(b). The z row of the optimal tableau from Table 12 on p. 289 has 1 under s_1 and s_2 and 0 under s_3, so the optimal solution of the dual is $y_1 = 1$, $y_2 = 1$, $y_3 = 0$.
(c). Plugging in $y_1 = 1$, $y_2 = 1$, $y_3 = 0$ in the constraints of the dual we get $2 + 1 + 0 \ge 3$, $1 + 1 + 0 \ge 2$, so this is a feasible solution of the dual problem (with slack variables $\eta_1 = \eta_2 = 0$). The objective value is $100 + 80 + 0 = 180$, which is the value of z in the optimal solution of the primal $x_1 = 20$, $x_2 = 60$, $x_3 = 20$ from that tableau.

6.7.3. We have $y_1 = 0.4$ and $y_2 = 1.4$, so the optimal z value is $\mathbf{y}^T\mathbf{b} = 0.5y_1 + 0.5y_2 = 0.9$.

6.7.5. If that tableau is correct, the dual problem has an optimal solution where $\eta_1 = 0$, $\eta_2 = 2$, $y_1 = 0$, $y_2 = 1$, and $z = 20/3$. Now the objective of the dual is $6y_1 + 10y_2$, which would be 10, not 20/3, so something is wrong. You might also notice that the first equation of the dual is $3y_1 + 6y_2 - \eta_1 = 4$; plugging in the above values gives $0 + 6 - 0 = 4$, which is also wrong.

E.1. If the first constraint is $x_1 - x_2 \le 0$, one way to make it really obvious that the primal problem is infeasible would be to take the second constraint as $x_1 - x_2 \ge 1$, i.e. $-x_1 + x_2 \le -1$. Then the constraints of the dual would be $y_1 - y_2 \ge c_1$ and $-y_1 + y_2 \ge c_2$. Since the latter is equivalent to $y_1 - y_2 \le -c_2$, the dual problem will also be infeasible if $-c_2 < c_1$. So e.g. with $c_1 = 1$ and $c_2 = 1$, the primal problem is

$$\begin{array}{lllll} \max & x_1 & +x_2 & & \\ \text{s.t.} & x_1 & -x_2 & \le & 0 \\ & -x_1 & +x_2 & \le & -1 \\ & x_1, x_2 \ge 0 & & & \end{array}$$

There are many other correct answers to this problem.