Math 340: Answers to Assignment 2

4.4.2. The feasible region is shown in Figure 4 on page 61. The feasible region has three extreme points: $(0,14)$, $(3.6,1.4)$ and $(12,0)$.

The extreme point $(0,14)$ is on the line $7x_1+2x_2=28$ but not on the line $2x_1+12x_2=24$, so this must correspond to a basic solution in which the surplus variable e_2 for the second constraint is basic. Moreover, x_2 has a nonzero value, so it must be basic. There must be two basic variables because there are two constraints, so the other variables e_1 and x_1 are nonbasic. Similarly, $(3.6,1.4)$ corresponds to the basic feasible solution where x_1 and x_2 are basic, e_1 and e_2 nonbasic, and $(12,0)$ corresponds to the basic feasible solution where x_1 and e_1 are basic and x_2 and e_2 are nonbasic.

4.5.3. The initial tableau is

z	x_1	x_2	x_3	s_1	s_2	s_3	
1	−2	1	−1	0	0	0	$0=z$
0	3	1	1	1	0	0	$60=s_1$
0	1	−1	2	0	1	0	$10=s_2$
0	1	1	−1	0	0	1	$20=s_3$

The basic solution is feasible, so Phase II can begin. Using the most-negative-entry pivoting rule, x_1 enters and s_2 leaves, the minimum ratio being 10.

z	x_1	x_2	x_3	s_1	s_2	s_3	
1	0	−1	3	0	2	0	$20=z$
0	0	4	−5	1	−3	0	$30=s_1$
0	1	−1	2	0	1	0	$10=x_1$
0	0	2	−3	0	−1	1	$10=s_3$

Now x_2 enters and s_3 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	
1	0	0	3/2	0	3/2	1/2	$25=z$
0	0	0	1	1	−1	−2	$10=s_1$
0	1	0	1/2	0	1/2	1/2	$15=x_1$
0	0	1	−3/2	0	−1/2	1/2	$5=x_2$

The optimal solution is $x_1=15$, $x_2=5$, $x_3=0$, $s_1=10$, $s_2=s_3=0$, $z=25$.

4.8.5. The initial tableau is

z	x_1	x_2	s_1	s_2	rhs		
1	−1	−2	0	0	0	=	z
0	−1	1	1	0	2	=	s_1
0	−2	1	0	1	1	=	s_2

x_2 enters; s_2 leaves, having the minimum ratio 1.

z	x_1	x_2	s_1	s_2	rhs		
1	−5	0	0	2	2	=	z
0	1	0	1	−1	1	=	s_1
0	−2	1	0	1	1	=	x_2

x_1 enters; s_1 leaves, having the only positive entry in the x_1 column.

z	x_1	x_2	s_1	s_2	rhs		
1	0	0	5	−3	7	=	z
0	1	0	1	−1	1	=	x_1
0	0	1	2	−1	3	=	x_2

s_2 enters, but there are no ratios to calculate, so the problem is unbounded.

4.11.4. "Ties are broken in favor of lower numbered rows" means our normal pivoting rules. The initial tableau is

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	3	−1	6	0	0	0	0	0	=	z
0	9	1	−9	−2	1	0	0	0	=	s_1
0	1	1/3	−2	−1/3	0	1	0	0	=	s_2
0	−9	−1	9	2	0	0	1	1	=	s_3

x_2 is the only candidate to enter; s_1 leaves (tied for least ratio 0, so lower-numbered row 1 wins over row 2).

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	12	0	−3	−2	1	0	0	0	=	z
0	9	1	−9	−2	1	0	0	0	=	x_2
0	−2	0	1	1/3	−1/3	1	0	0	=	s_2
0	0	0	0	0	1	0	1	1	=	s_3

x_3 has the most negative entry in the z row; s_2 leaves with the only ratio.

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	6	0	0	−1	0	3	0	0	=	z
0	−9	1	0	1	−2	9	0	0	=	x_2
0	−2	0	1	1/3	−1/3	1	0	0	=	x_3
0	0	0	0	0	1	0	1	1	=	s_3

x_4 is the only candidate to enter; x_2 leaves (with lower-numbered row).

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	−3	1	0	0	−2	12	0	0	=	z
0	−9	1	0	1	−2	9	0	0	=	x_4
0	1	−1/3	1	0	1/3	−2	0	0	=	x_3
0	0	0	0	0	1	0	1	1	=	s_3

x_1 has the most negative entry in the z row; x_3 leaves, with the only ratio.

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	0	0	3	0	−1	6	0	0	=	z
0	0	−2	9	1	1	−9	0	0	=	x_4
0	1	−1/3	1	0	1/3	−2	0	0	=	x_1
0	0	0	0	0	1	0	1	1	=	s_3

s_1 has the only negative entry in the z row: x_4 leaves (lower-numbered row with ratio 0)

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	0	−2	12	1	0	−3	0	0	=	z
0	0	−2	9	1	1	−9	0	0	=	s_1
0	1	1/3	−2	−1/3	0	1	0	0	=	x_1
0	0	2	−9	−1	0	9	1	1	=	s_3

s_2 has the most negative entry in the z row; x_1 leaves with the minimum ratio 0. But this takes us back to the original tableau (with basis s_1, s_2, s_3, so cycling has occurred.

4.11.5. Using Bland's Rule, the first pivot is the same (only one candidate to enter; for leaving, s_1 comes before s_2). The second pivot is the same (for entering, x_3 comes before x_4; only one candidate to leave). The third pivot is the same (only one candidate to enter; for leaving, x_2 comes before x_3). The fourth pivot is the same (for entering, x_1 comes before s_1; only one candidate to leave). On the fifth pivot, s_1 is the only candidate to enter, but x_1 leaves instead of x_4. The result is

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	3	−1	6	0	0	0	0	0	=	z
0	−3	−1	6	1	0	−3	0	0	=	x_4
0	3	−1	3	0	1	−6	0	0	=	s_1
0	−3	1	−3	0	0	6	1	1	=	s_3

Now x_2 enters, and s_3 leaves in a nondegenerate pivot.

z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs		
1	0	0	3	0	0	6	1	1	=	z
0	−6	0	3	1	0	3	1	1	=	x_4
0	0	0	0	0	1	0	1	1	=	s_1
0	−3	1	−3	0	0	6	1	1	=	x_2

This is optimal: $x_1 = 0$, $x_2 = 1$, $x_3 = 0$, $x_4 = 1$, $s_1 = 1$, $s_2 = s_3 = 0$. Cycling was avoided.

E.1. The initial tableau is

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	−2	−3	−3	0	0	0	0	=	z
0	3	2	0	1	0	0	60	=	s_1
0	−1	1	4	0	1	0	10	=	s_2
0	2	−2	5	0	0	1	5	=	s_3

x_2 enters and s_2 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	−5	0	9	0	3	0	30	=	z
0	5	0	−8	1	−2	0	40	=	s_1
0	−1	1	4	0	1	0	10	=	x_2
0	0	0	13	0	2	1	25	=	s_3

x_1 enters and s_1 leaves.

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs		
1	0	0	1	1	1	0	70	=	z
0	1	0	$-8/5$	$1/5$	$-2/5$	0	8	=	x_1
0	0	1	$12/5$	$1/5$	$3/5$	0	18	=	x_2
0	0	0	13	0	2	1	25	=	s_3

This is optimal. The optimal solution is $x_1 = 8$, $x_2 = 18$, $x_3 = s_1 = s_2 = 0$, $s_3 = 25$, $z = 70$.

E.2. Cycling is rare in problems that arise in practice, although it does occur occasionally. On the other hand, from the theoretical point of view, the fact that cycling can be avoided is very important, for two reasons:

(1) We could not call the Simplex Method an algorithm if it was not guaranteed to terminate.

(2) We used the fact that the Simplex Method terminates to prove that every Linear Programming problem falls into one of the three categories: infeasible, unbounded, or having an optimal solution which is basic.