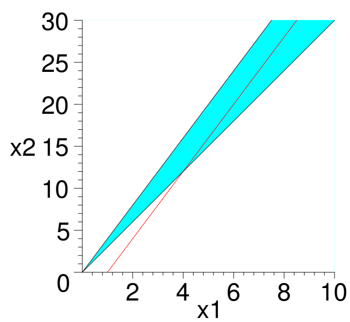
Math 340 sec. 101: Answers to Assignment 1

3.3.10(a). The main point is that he can pay for the dollars he buys with the francs he buys, and vice versa. So (if he starts out with nothing), at 12:01 he has $x_1 - .25x_2$ dollars and $x_2 - 3x_1$ francs. The only requirements are that both of these must be nonnegative (i.e. he must be able to pay for his purchases). So the problem is

$$\begin{array}{ll} \max & x_1 - .25x_2 \\ \text{s.t.} & x_1 - .25x_2 \geq 0 \\ & -3x_1 + x_2 \geq 0 \\ & x_1, x_2 \geq 0 \end{array}$$

(b). The graph below shows part of the feasible region, which is unbounded, and one of the isoprofit lines. To increase the profit, we want to slide the isoprofit line to the right. No matter how far we go, it will still intersect the feasible region. Thus the problem is unbounded.



3.4.3(a). Let x_1 and x_2 be the number of pounds of the Food 1 and Food 2 respectively that Peg buys. The problem is

$$\begin{array}{ll} \min & 7x_1 + x_2 \\ \text{s.t.} & 3x_1 + x_2 \geq 12 \\ & x_1 + x_2 \geq 6 \\ & x_1, x_2 \geq 0 \end{array}$$

Solving the problem (either graphically or with LINDO), the optimal solution is $x_1 = 0$, $x_2 = 12$. This results in 12 units of vitamin C, which is an excess of 6 units.

(b). Solving the two equations $3x_1 + x_2 = 12$ and $x_1 + x_2 = 6$, we get $x_1 = 3$ and $x_2 = 3$. This costs \$24, while the optimal solution costs only \$12. Even without calculating it, we should have known this would be more expensive, because it would be a feasible solution of the original problem but not the optimal solution.

3.5.5. The variables x_1 to x_6 represent the numbers of workers in each of the 6 possible pairs of shifts:

x_1	12 am - 6 am, 6 am - 12 pm	x_2	12 am - 6 am, 12 pm - 6 pm
x_3	12 am - 6 am, 6 pm - 12 am	x_4	6 am - 12 pm, 12 pm - 6 pm
x_5	6 am - 12 pm, 6 pm - 12 am	x_6	12 pm - 6 pm, 6 pm - 12 am

where only x_2 and x_5 have non-consecutive shifts. The problem is

$$
\begin{array}{ll}
\text{min} & 12x_1 + 18x_2 + 12x_3 + 12x_4 + 18x_5 + 12x_6 \\
\text{s.t.} & x_1 + x_2 + x_3 \ge 15 \\
& x_1 + x_4 + x_5 \ge 5 \\
& x_2 + x_4 + x_6 \ge 12 \\
& x_3 + x_5 + x_6 \ge 6 \\
\text{all } x_i \ge 0 &
\end{array}
$$

Actually the objective should all be multiplied by 12 to get the total wages per day, since the \$18 and \$20 are per hour but each person works 12 hours. But these numbers are more convenient.

According to LINDO, an optimal solution, with an objective value of 276, is $x_1 = 5$, $x_2 = 8$, $x_3 = 2$, $x_4 = x_5 = 0$, $x_6 = 4$. It turns out that this is not the only optimal solution: another is $x_1 = 1$, $x_2 = 8$, $x_3 = 6$, $x_4 = 4$, $x_5 = x_6 = 0$. LINDO might give you either of these.

3.8.2. Let J_i be the number of pounds of grade i oranges used for juice, and B_i the number of pounds of grade i oranges sold in bags, where i is 6 or 9. We have the constraints $J_6 + B_6 \le 120000$ and $J_9 + B_9 \le 100000$, giving the total amounts available of each grade. The constraint on the quality of oranges in bags can be written as $6B_6 + 9B_9 \ge 7(B_6 + B_9)$, i.e. $-B_6 + 2B_9 \ge 0$. Similarly, the constraint on the quality of juice can be written as $6J_6 + 9J_9 \ge 8(J_6 + J_9)$ or $-2J_6 + J_9 \ge 0$. The objective is to maximize profit, which is $.45(J_6 + J_9) + .3(B_6 + B_9)$. So the LINDO file is:

```
max .45 J6 + .45 J9 + .3 B6 + .3 B9
st
J6 + B6 <= 120000
J9 + B9 <= 100000
-B6 + 2 B9 >= 0
-2 J6 + J9 >= 0
```

LINDO's optimal solution is

```
OBJECTIVE FUNCTION VALUE
1)       78000.00
VARIABLE         VALUE          REDUCED COST
J6      26666.666016          0.000000
J9      53333.332031          0.000000
B6      93333.335938          0.000000
B9      46666.667969          0.000000
ROW   SLACK OR SURPLUS     DUAL PRICES
2)          0.000000          0.150000
3)          0.000000          0.600000
4)          0.000000         -0.150000
5)          0.000000         -0.150000
```

Thus of the Grade 6 oranges, 26667 lb are used for juice and the remaining 93333 lb sold in bags, while 53333 lb of the Grade 9 oranges are used for juice and 46667 lb sold in bags.

Oil Refinery Problem.

I used the following variables. Each represents an amount in barrels per day.

`crude1, crude2`	crude oils
`ln, mn, hn`	light, medium, heavy naphthas
`lo, ho, res`	light and heavy oil and residuum
`lnref, mnref, hnref`	naphthas used in reforming
`rfg`	reformed gasoline

`locr, hocr`	light and heavy oils used in cracking
`cro, crg`	cracked oil and gasoline
`lnprem, mnprem, hnprem`	naphthas used in premium gasoline
`crgprem rfgprem`	cracked and reformed gasoline used in premium gasoline
`lnreg, mnreg, hnreg`	naphthas used in regular gasoline
`crgreg rfgreg`	cracked and reformed gasoline used in regular gasoline
`lojf, hojf, crojf, resjf`	oils and residuum used for jet fuel
`lofo, hofo, crofo, resfo`	oils and residuum used for fuel oil
`reslub`	residuum used for lube oil
`prem, reg, jf, fo, lube`	final products

Here was my LINDO file:

```
max
! revenue from sales
35 prem + 34 reg + 29 jf + 28.5 fo + 26 lube
! - crude oil costs
-24 crude1 - 26 crude2
! - cracking and reforming costs
- locr - hocr  - lnref - mnref - hnref
st
! yields from distillation
distln) ln -.1 crude1 -.15 crude2 = 0
distmn) mn -.2 crude1 -.25 crude2 = 0
disthn) hn -.2 crude1 -.18 crude2 = 0
distlo) lo -.12 crude1 -.08 crude2 = 0
distho) ho -.20 crude1 -.19 crude2 = 0
distres) res-.13 crude1 -.12 crude2 = 0
! material balance for napthas
lnmb) ln - lnreg - lnprem - lnref = 0
mnmb) mn - mnreg - mnprem - mnref = 0
hnmb) hn - hnreg - hnprem - hnref = 0
! yield from reforming
refgas) rfg - .6 lnref - .52 mnref -.45 hnref = 0
! material balance for light and heavy oils
lomb) lo - locr - lojf - lofo = 0
homb) ho - hocr - hojf - hofo = 0
! yields from cracking
crackoil) cro - .68 locr - .75 hocr = 0
crackgas) crg - .28 locr - .20 hocr = 0
! material balance for cracked oil and residuum
cromb) cro - crojf - crofo = 0
resmb) res - resjf - resfo - reslub = 0
! material balance for reformed and cracked gas
rfgmb) rfg - rfgreg - rfgprem = 0
crgmb) crg - crgreg - crgprem = 0
! material balance for regular and premium gasoline
regmb) reg - lnreg - mnreg - hnreg - rfgreg - crgreg = 0
premb) prem - lnprem - mnprem - hnprem - rfgprem - crgprem = 0
! octane-number constraints for regular and premium gasoline
regoct) 90 lnreg + 80 mnreg + 70 hnreg + 115 rfgreg + 105 crgreg
```

```
- 84 reg >= 0
premoct) 90 lnprem + 80 mnprem + 70 hnprem + 115 rfgprem + 105 crgprem
- 94 prem >= 0
! material balance for jet fuel
jfmb) jf - lojf - hojf - crojf - resjf = 0
!vapour-pressure constraints on jet fuel
jfvp1) 10 lojf + 6 hojf + 15 crojf + .5 resjf - 8 jf >= 0
jfvp2) 10 lojf + 6 hojf + 15 crojf + .5 resjf - 10 jf <= 0
! fuel oil production
folo) 18 lofo - 10 fo = 0
focr) 18 crofo - 4 fo = 0
foho) 18 hofo - 3 fo = 0
fore) 18 resfo - fo = 0
! lube oil yield
lubep) lube - .5 reslub = 0
! availabilities of crude oil
cr1bd) crude1 <= 20000
cr2bd) crude2 <= 30000
! capacities for distillation, reforming and cracking
distbd) crude1+crude2 <= 45000
refbd) lnref + mnref + hnref <= 10000
crackbd) locr + hocr <= 8000
! bounds on lube production
lubelo) lube >= 500
lubehi) lube <= 1000
end
```

LINDO's solution was:

```
LP OPTIMUM FOUND AT STEP     21
OBJECTIVE FUNCTION VALUE
1)      160553.6
```

VARIABLE	VALUE	REDUCED COST
PREM	0.000000	0.000000
REG	24664.259766	0.000000
JF	15519.264648	0.000000
FO	0.000000	1.300012
LUBE	500.000000	0.000000
CRUDE1	20000.000000	0.000000
CRUDE2	25000.000000	0.000000
LOCR	3296.213867	0.000000
HOCR	4703.786133	0.000000
LNREF	0.000000	5.719140
MNREF	0.000000	3.138820
HNREF	3089.887695	0.000000
LN	5750.000000	0.000000
MN	10250.000000	0.000000
HN	8500.000000	0.000000
LO	4400.000000	0.000000
HO	8750.000000	0.000000

RES	5600.000000	0.000000
LNREG	5750.000000	0.000000
LNPREM	0.000000	5.040250
MNREG	10250.000000	0.000000
MNPREM	0.000000	2.520125
HNREG	5410.112305	0.000000
HNPREM	0.000000	0.000000
RFG	1390.449463	0.000000
LOJF	1103.786133	0.000000
LOFO	0.000000	0.000000
HOJF	4046.213867	0.000000
HOFO	0.000000	0.000000
CRO	5769.265137	0.000000
CRG	1863.697144	0.000000
CROJF	5769.265137	0.000000
CROFO	0.000000	0.000000
RESJF	4600.000000	0.000000
RESFO	0.000000	0.000000
RESLUB	1000.000000	0.000000
RFGREG	1390.449463	0.000000
RFGPREM	0.000000	11.340564
CRGREG	1863.697144	0.000000
CRGPREM	0.000000	8.820438

ROW	SLACK OR SURPLUS	DUAL PRICES
DISTLN)	0.000000	38.228981
DISTMN)	0.000000	31.180679
DISTHN)	0.000000	24.132380
DISTLO)	0.000000	29.834795
DISTHO)	0.000000	28.165205
DISTRES)	0.000000	25.869518
LNMB)	0.000000	-38.228981
MNMB)	0.000000	-31.180679
HNMB)	0.000000	-24.132380
REFGAS)	0.000000	55.849731
LOMB)	0.000000	-29.834795
HOMB)	0.000000	-28.165205
CRACKOIL)	0.000000	31.921782
CRACKGAS)	0.000000	48.801430
CROMB)	0.000000	-31.921782
RESMB)	0.000000	-25.869518
RFGMB)	0.000000	-55.849731
CRGMB)	0.000000	-48.801430
REGMB)	0.000000	-25.205725
PREMB)	0.000000	-7.564848
REGOCT)	0.000000	-0.704830
PREMOCT)	0.000000	-0.452818
JFMB)	0.000000	25.660820
JFVP1)	0.000000	-0.417397

```
   JFVP2)     31038.529297            0.000000
    FOLO)         0.000000           -1.657489
    FOCR)         0.000000           -1.773432
    FOHO)         0.000000           -1.564734
    FORE)         0.000000           -1.437196
   LUBEP)         0.000000           51.739037
   CR1BD)         0.000000            0.745904
   CR2BD)      5000.000000            0.000000
  DISTBD)         0.000000            2.715860
   REFBD)      6910.112305            0.000000
 CRACKBD)         0.000000            4.536417
  LUBELO)         0.000000          -25.739038
  LUBEHI)       500.000000            0.000000
```

According to this, we buy all the available Crude I (20000 barrels) and 25000 barrels of Crude 2 (reaching the limit on distillation). All 5750 barrels of light naphtha, all 10250 barrels of medium naphtha, and about 5410 of the 8500 barrels of heavy naphtha produced go into regular gasoline. The other 3090 barrels of heavy naphtha are used to make reformed gasoline (about 1390 barrels), which then goes into regular gasoline. Of the 4400 barrels of light oil, 3296 barrels go into cracking and the other 4046 barrels go into jet fuel. Of the 8750 barrels of heavy oil, 4704 barrels go into cracking and the other 4046 barrels go into jet fuel. The cracking process produces 5760 barrels of cracked oil, which all goes into jet fuel, and 1864 barrels of cracked gasoline, which all goes into regular gasoline. Of the 5600 barrels of residuum, 4600 barrels go into jet fuel and the remaining 1000 barrels into lube oil. The end products are 24664 barrels of regular gasoline, 15519 barrels of jet fuel and 500 barrels of lube oil.