Duality and Tableaus

Consider the (primal) linear programming problem

$$\begin{array}{ll} \text{maximize} & z = 2\,x_1 + 3\,x_2 + 4\,x_3 \\ \text{subject to} & 5\,x_1 + 6\,x_2 + 7\,x_3 \le 8 \\ & 9\,x_1 + 10\,x_2 + 11\,x_3 \le 12 \\ & x_1, x_2, x_3 \ge 0 \end{array}$$

The dual of this problem is

$$\begin{array}{ll} \text{minimize} & w = 8y_1 + 12y_2 \\ \text{subject to} & 5y_1 + 9y_2 \ge 2 \\ & 6y_1 + 10y_2 \ge 3 \\ & 7y_1 + 11y_2 \ge 4 \\ & y_1, y_2 \ge 0 \end{array}$$

Initial tableau for the primal problem:

z	x_1	x_2	x_3	s_1	s_2	rhs		
1	-2	-3	-4	0	0	0	=	z
0	5	6	7	1	0	8	=	s_1
0	9	10	11	0	1	12	=	s_2

Initial tableau for the dual problem:

w	y_1	y_2	η_1	η_2	η_3	rhs		
1	8	12	0	0	0	0	=	w
0	-5	-9	1	0	0	-2	=	η_1
0	-6	-10	0	1	0	-3	=	η_2
0	-7	-11	0	0	1	-4	=	η_3

Primal tableau after pivot (x_3 enters, s_2 leaves):

z	x_1	x_2	x_3	s_1	s_2	rhs		
1	14/11	7/11	0	0	4/11	48/11	=	z
0	−8/11	−4/11	0	1	−7/11	4/11	=	s_1
0	9/11	10/11	1	0	1/11	12/11	=	x_3

Dual tableau after pivot (y_2 enters, η_3 leaves):

w	y_1	y_2	η_1	η_2	η_3	rhs		
1	4/11	0	0	0	12/11	−48/11	=	w
0	8/11	0	1	0	−9/11	14/11	=	η_1
0	4/11	0	0	1	−10/11	7/11	=	η_2
0	7/11	1	0	0	−1/11	4/11	=	y_2

Primal-Dual Correspondence

Primal	Dual
decision variable x_j	slack variable η_j
slack variable s_i	decision variable y_j
objective coefficients	rhs constants
rhs constants	objective coefficients
basic	nonbasic
nonbasic	basic
ordinary variable (≥ 0)	ordinary variable(≥ 0)
equality constraint	URS variable
URS variable	equality constraint
optimal	optimal

If the primal problem is unbounded, the dual is infeasible. If the dual problem is unbounded, the primal is infeasible. It is also possible for both primal and dual problems to be infeasible.