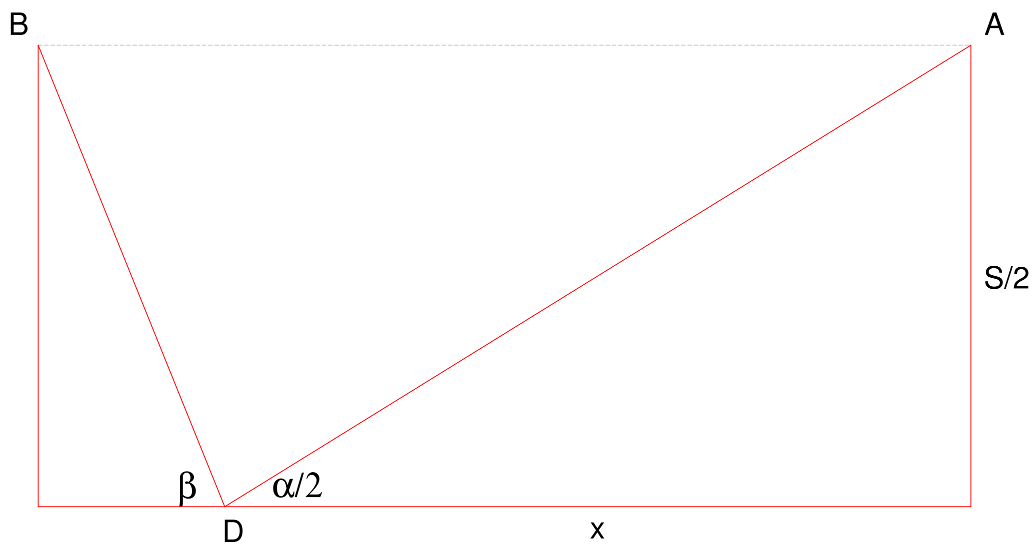
Zebra Danio
Calculus to the Rescue

"Suppose", says Danny to himself, "I swim at an angle β from the negative x axis, where $0 < \beta \le \pi/2$. To reach $y = S/2$ I must swim a distance of $S/2\csc(\beta)$. Here's a diagram: I'm at D, the edge of the shark is at A, and I have to get to B before the shark does."



"If I swim at speed w, this will take time $S\csc(\beta)/(2w)$. If $0 < \beta < \pi/2$, $\csc(\beta) > 1$, so that will take longer than if I went directly in the y direction with $\beta = \pi/2$; but since it also takes me a distance $S\cot(\beta)/2$ to the left, the shark will also have longer to go, namely $x + S\cot(\beta)/2$. The shark's speed is v, so this takes time $x/v + S\cot(\beta)/(2v)$. I will survive if $S\csc(\beta)/(2w) < x/v + S\cot(\beta)/(2v)$, i.e.

$$-\frac{2x}{S} < \cot(\beta) - \frac{v\csc(\beta)}{w}$$

"It will be useful to write $w = cv$, i.e. c is the ratio of my speed to the shark's speed, so the condition becomes

$$-\frac{2x}{S} < \cot(\beta) - \frac{\csc(\beta)}{c}$$

"Is this possible? I want to choose β in the interval $0 < \beta \le \pi/2$ to maximize

$$f(\beta) = \cot(\beta) - \frac{\csc(\beta)}{c}$$

which will ensure that I survive (I hope) with as much time to spare as possible. Differentiating, I get

$$f'(\beta) = -\csc^2(\beta) + \frac{\csc(\beta)\cot(\beta)}{c} = \frac{\cos(\beta) - c}{c\sin^2(\beta)}$$

"The only critical point in the interval is $\beta = \arccos(c)$. I'll call this β_0. Note that this is defined since $0 < c < 1$. It's the only critical point in the interval, and I'm pretty sure it's a maximum rather than a minimum (if I wasn't swimming for my life right now I might prove that using the Second Derivative Test). Note that

$$\sin(\arccos(c)) = \sqrt{1-c^2}, \qquad \tan(\arccos(c)) = \frac{\sqrt{1-c^2}}{c}$$

so

$$f(\beta_0) = \frac{c}{\sqrt{1-c^2}} - \frac{1}{c\sqrt{1-c^2}} = -\frac{\sqrt{1-c^2}}{c}$$

"Using this optimal β_0, I will survive if $-\dfrac{2x}{S} < -\dfrac{\sqrt{1-c^2}}{c}$, i.e.

$$\frac{2x}{S} > \frac{\sqrt{1-c^2}}{c}$$

"Oops: now I have a problem. I know my own speed w, but I don't know the shark's speed v, so I don't know c. Or do I? What I do know are the visual angle α and the rate of change of that angle. Let's look again at the formula for the visual angle, which I'll write in slightly different form:

$$\cot(\alpha/2) = \frac{2x}{S}$$

"So from the initial visual angle $\alpha = 0.6$ I can compute $2x/S = \cot(0.3) = 3.232728144$ approximately. Differentiating, I get

$$-\frac{\csc^2(\alpha/2)}{2}\frac{d\alpha}{dt} = \frac{2}{S}\frac{dx}{dt}$$

"Initially, when I was at rest, I had $dx/dt = -v$ and $d\alpha/dt = -0.06$. Plugging in these, I get

$$-0.03\csc^2(\alpha/2) = -\frac{2v}{S}$$

"But now I'm moving at speed $w = cv$, so I have $dx/dt = w - v = -(1-c)v$ and $d\alpha/dt = -0.04$. Plugging in these, I get

$$-0.02\csc^2(\alpha/2) = -\frac{2(1-c)v}{S}$$

"These measurements were taken at slightly different times, but not enough for α to have changed appreciably. So I can compute

$$1 - c = \frac{0.02}{0.03} = \frac{2}{3}, \qquad c = 1 - \frac{2}{3} = \frac{1}{3}$$

So $\beta_0 = \arccos(1/3) = 1.230959417$ radians or about 70.53 degrees. That's the direction I need to go. And since

$$\frac{\sqrt{1-c^2}}{c} = \frac{\sqrt{1-(1/3)^2}}{1/3} = \sqrt{8} < 3.232728144...$$

"Yes! I will survive!"