Write clearly and legibly, in complete sentences. You may discuss the homework with other students, but the final write-up must be your own. If your solution uses any results not introduced in class, state the result clearly and provide either a reference or a proof.

1. (10 points) Write out in detail the proof of the first part of Proposition 1.2 (d). ($\mathcal{B}_\mathbb{R}$ is generated by the open rays $\mathcal{E}_\mathbb{R} = \{(a, \infty) : a \in \mathbb{R}\}$. (Do not use the other parts of the proposition, unless you also prove those in detail.)

2. 10 points Let $E = [0, 1]^2 \setminus \mathbb{Q}^2$ in $\mathbb{R}^2$. Prove that $E$ is a $G_\delta$ set (see the definitions after Lemma 1.1 in the textbook).

3. (10 points) Section 1.2, Problem 4. Prove that an algebra $\mathcal{A}$ is a $\sigma$-algebra if and only if it is closed under countable increasing unions (if $E_1, E_2, \ldots \in \mathcal{A}$ and $E_1 \subset E_2 \subset \ldots$, then $\bigcup_{i=1}^{\infty} E_i \in \mathcal{A}$).

4. Section 1.3, Problem 8: Let $(X, \mathcal{M}, \mu)$ be a measure space, and let $\{E_j\}_{j=1}^{\infty} \subset \mathcal{M}$.
   (a) (10 points) Prove that $\mu(\lim \inf E_j) \leq \lim \inf \mu(E_j)$
   (b) (10 points) Prove that if $\mu(\bigcup_{i=1}^{\infty} E_j) < \infty$, then $\mu(\lim \sup E_j) \geq \lim \sup \mu(E_j)$.
(For the definitions of lim sup and lim inf, see Section 0.1.)