January 18, 2006

Math 318 Assignment 2: Due Wednesday, January 25 at start of class

I. Problems to be handed in:

1. Ross p20 #41. *Explanation:* Every rat has two genes, each of which may be a "black gene" or a "brown gene". Black genes "dominate", which means that if a rat has any black gene then it will be black in colour. So a rat can be "pure black" (two black genes), or "hybrid black" (one black and one brown gene), or brown (two brown genes). When two rats mate, each of their offspring independently inherits one randomly chosen gene from each parent.

2. Let A, B and C be independent events. Show that A and $B \cap C$ are independent. Show that A and $B \cup C$ are independent.

3. Ross p87 #13

4. The following gambling game is called wheel of fortune or chuck-a-luck. A player chooses a number from 1 to 6. Three fair dice are rolled, and if the player's number appears k times (for $k = 1, 2, 3$) then the player wins \$k. If the number does not appear, the player loses \$1. Find the player's expected profit.

5. (St Petersburg paradox with limited casino wealth). A gambler tosses a fair coin until the first time it comes up Heads. She wins $\$2^{n-1}$ if the number of tosses is n, unless this amount exceeds $\$2^{30}$, in which case the casino goes bankrupt and she wins "only" $\$2^{30}$.

   (a) Find the gambler's expected winnings.

   (b) If the gambler pays the answer in (a) as an entry fee, what is the probability she makes a profit?

II. Recommended problems: These provide additional practice but are not to be handed in. Starred problems have solutions in the text, and answers are given otherwise.

A. Is it possible for two events to be disjoint and also independent? If so, when?

B. Three sections of a class contain 20, 30 and 50 students respectively. (a) If a section is chosen at random, what is its expected size? (b) If a student is chosen at random, what is his/her expected section size? [$33\frac{1}{3}$, 38].

C. Ross p17: 23,25*,30*,33[9],40*. p85: 1[$\{rr, oo, bb, ro, ob, br\}$; $0, 1, 2$; $\frac{1}{2}$],12[$11/3^5$],16*,32[$1 - e^{-\frac{1}{2}}$, $e^{-\frac{1}{2}}/2$, $1 - 3e^{-\frac{1}{2}}/2$].

Quote of the week: *The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the true logic for this world is the calculus of Probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man's mind.* James Clerk Maxwell (1850)