MATH 307 Lecture 13

Quiz 2 today.

CHAPTER II
Subspaces, Bases, and Dimensions

II.1. Vector spaces and subspaces
What is a vector and what kind of properties we have been using?

A $n$-tuple $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ is a vector where $x_i \in \mathbb{R}$.

The set of all such $n$-tuples satisfies the following properties:
- If we add two n-tuples, it is again an n-tuple

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{bmatrix}
+ \begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_n
\end{bmatrix}
= \begin{bmatrix}
  x_1 + y_1 \\
  x_2 + y_2 \\
  \vdots \\
  x_n + y_n
\end{bmatrix} \in \mathbb{R}^n.
\]

- Multiplication of n-tuple by a scalar $\alpha \in \mathbb{R}$

\[
\alpha \begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{bmatrix}
= \begin{bmatrix}
  \alpha x_1 \\
  \alpha x_2 \\
  \vdots \\
  \alpha x_n
\end{bmatrix} \in \mathbb{R}^n.
\]

- $\mathbb{R}$, the set of scalars also satisfies $\forall \alpha, \beta \in \mathbb{R}$

$\alpha + \beta \in \mathbb{R}, \quad \alpha \beta \in \mathbb{R}$. 
By abstraction of these properties, the definition of a vector space $V$ over the scalar set $F$ is as follows: