MATH 307 Lecture 10

HW 2 (due: Mon Feb 3)
Quiz 2 on Wed Feb 5

con4 iii)

\[ \Rightarrow P_{j+1}''(x_{j+1}) = z_{j+1} \]
\[ \left\{ \begin{array}{l}
\quad P_j''(x_{j+1}) = z_{j+1} \\
\quad P_{n-1}(x_i) = z_n = 0
\end{array} \right\} \quad 2 \text{ equations} \]

iv) \[ P_j''(x_i) = z_i = 0 \]
\[ \left\{ \begin{array}{l}
\quad P_{n-1}(x_i) = z_n = 0
\end{array} \right\} \quad 2 \text{ equations} \]

v) \[ P_j'(x_{j+1}) = P_{j+1}'(x_{j+1}) \]
\[ \left( \frac{x_{j+2} - x_j}{6} \right) z_j + \left( \frac{x_{j+2} - x_j}{3} \right) z_{j+1} + \left( \frac{x_{j+2} - x_j}{6} \right) z_{j+2} \]
\[ = \frac{y_{j+2} - y_{j+1}}{x_{j+2} - x_{j+1}} - \frac{y_{j+1} - y_j}{x_{j+1} - x_j} \quad (\text{for all } j=1, \ldots, n-2) \]
\[ \Rightarrow \text{ check this!} \]
\[ \Rightarrow n-2 \# \text{ of equations!} \]
\[
S = \begin{pmatrix}
0 & 0 & \cdots & 0 & \frac{x_1 - x_n}{x - x_1} \\
0 & 0 & \cdots & 0 & \frac{x_2 - x_1}{x - x_2} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & \frac{x_n - x_1}{x - x_n}
\end{pmatrix}
\]
Total: n # of equations to find $z_1, z_2, \ldots, z_n$

Solve: $Sa = b$

where $a = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$
The second method for cubic splines, involving the equation \( S \mathbf{a} = \mathbf{b} \).
Not only is the matrix \( S \) well-conditioned (low condition number), but \( S \mathbf{a} = \mathbf{b} \) can be solved efficiently due to the tridiagonal structure.

I.3. Finite difference approximation.
- Numerical method to solve differential equations.

Motivation?
Many physical phenomena are based on differential equations.
Ex: \( f''(x) = 1 \) for all \( x \).

Find \( f(x) \):

\[ \Rightarrow f'(x) = x + C_1 \quad (+) \]

\[ \Rightarrow f(x) = \frac{1}{2}x^2 + C_1x + C_2 \quad (*) \]

for arbitrary constants \( C_1, C_2 \)

(Infinitely many solutions)

Usually, additional constraints are imposed to make \( C_1, C_2 \) uniquely determined.

1. For example, \( f(0) = 1 \)

\[ \Rightarrow f(0) = 0 + C_1 \cdot 0 + C_2 = C_2 = 1 \]

Then, from (*) and above two conditions,

\[ f(0) = \frac{1}{2} \cdot 0^2 + C_1 \cdot 0 + C_2 = C_2 = 1 \]

\[ f(1) = \frac{1}{2} \cdot 1^2 + C_1 \cdot 1 + C_2 \]

\[ = \frac{1}{2} + C_1 + 1 = 2 \]

\[ \Rightarrow C_1 = \frac{1}{2} \]