Math 421/510: Topic List for Midterm Test (with Folland section references)

1. Banach Spaces (5.1-5.3)
   (a) norms and completeness
   (b) example: $C([0,1]), \|\cdot\|_\infty$
   (c) linear maps and their boundedness/continuity
   (d) linear functionals and the dual space
   (e) Hahn-Banach theorem
   (f) embedding of X into X^{**} and reflexivity
   (g) Baire Category theorem
   (h) open mapping, closed graph & uniform boundedness theorems
2. L^p Spaces (6.1-6.2)
   (a) definitions, Hölder and triangle inequalities
   (b) characterization of the dual space $(L^p)^*$
3. Hilbert Spaces (5.5)
   (a) inner products
   (b) example: L^2
   (c) orthogonality & orthogonal decomposition
   (d) orthonormal sets, Bessel inequality, orthonormal basis, Parseval's identity
4. Other Topologies on Vector Spaces (5.4)
   (a) very basics of point-set topology: open, closed & compact sets, convergence of sequences, continuity, topology generated by collections of sets/functions, product topology (from Folland 4.1,4.2,4.4)
   (b) weak and weak-* topologies & convergence on Banach spaces
   (c) Alaoglu's theorem
   (d) weak and strong operator topologies on $\mathcal{L}(X,Y)$
   (e) topologies generated by semi-norms; examples: L^1_{loc}, C^∞

(Feb 25)