MATH 420/507: Real Analysis I/ Measure Theory and Integration, 2018W1

Short list of key topics and notions

A Measure Theory:

- algebras, σ-algebras, Borel sets;
- measures (finite, σ-finite, complete) and their properties (e.g. continuity);
- premeasures, outer measures, measurables sets, Carathéodory's theorem;
- Lebesgue (and Lebesgue-Stieltjes) measure on $\mathbb{R}$, and its regularity;

B Integration:

- measurable and simple functions, approximation of measurable by simple;
- integration on L^+ and L^1;
- convergence theorems: MCT, Fatou, DCT;

C Convergence and Approximation

- approximation in L^1 by simple functions (by continuous, for $(\mathbb{R}, \mathcal{L}, m)$);
- convergence: pointwise, a.e., uniform, L^1, in-measure;
- relations between these modes (including subsequential a.e., Egoroff);

D Product Measure

- product σ-algebras, rectangles, product measure;
- sections, slicing theorem, Fubini and Tonelli theorems;
- $(\mathbb{R}^n, \mathcal{B}_{\mathcal{R}^n}, m^n)$, and its regularity;

E Differentiation of Measures

- signed measures, Hahn decomposition, Jordan decomposition;
- mutual singularity, absolute continuity; $d\nu = f d\mu$;
- Lebesgue decomposition, Radon-Nikodym theorem;

F Differentiation on $\mathbb{R}^n$

- maximal function, maximal theorem;
- Lebesgue differentiation theorem, differentiation of Borel measures on $\mathbb{R}^n$;

G Differentiation on $\mathbb{R}$

- increasing and BV functions and their a.e. differentiability;
- NBV and complex Borel measures on $\mathbb{R}$;
- absolute continuity, and the FTC for Lebesgue integration;

H Introduction to L^p Spaces

- L^p, L^∞, Hölder's inequality, Minkowski inequality;