Math 401: List of Core Skills

A student sucessfully completing Math 401 (Spring 2012) should:

- **ODE BVPs**:
  - understand the Green's function: what equation it satisfies (involves adjoint operators and BCs), and how to represent solutions in terms of it
  - know how to find the GF by "solving on the left and right + matching"
  - know how to represent the GF by eigenfunction expansion
  - know how to find solvability conditions, and understand modified GFs
- **Steady-state (especially Laplace/Poisson) PDE**:
  - understand the Green's function: what equation it satisfies (including BCs), and how to represent solutions in terms of it
  - know how to find the GF using the free-space Green's function and the method of images
  - know how to represent the GF by eigenfunction expansion
  - understand the mean-value property and maximum principle for harmonic functions, and how to use the latter to prove uniqueness results
  - know how to find solvability conditions, and understand modified GFs
- **Time-dependent (especially heat and wave) PDE**:
  - understand the Green's function: what equation it satisfies (including BCs), and how to represent solutions in terms of it
  - know how to find the GF using the free-space Green's function and the method of images
  - know how to represent the GF by eigenfunction expansion
  - understand the maximum principle for the heat equation, and how to use it to prove uniqueness results
- **Eigenvalue problems (especially second-order)**:
  - know what the eigenvalue problem is, and the main properties of the eigenvalues and eigenfunctions of a second-order self-adjoint operator
  - know the various variational characterizations of the eigenvalues in terms of the Rayleigh quotient (including the min-max principle)
  - be able to find bounds on eigenvalues using: trial functions in the Rayleigh quotient; bounds on the coefficients; bounds on the goemetry
- **Calculus of variations**
  - be able to derive Euler-Lagrange equations for variational problems
  - be able to derive Euler-Lagrange equations for contrained variational problems, using Lagrange multipliers
  - be able to approximate solutions of variational problems using the Rayleigh-Ritz method
  - be able to approximate eigenvalues and eigenfunctions using the Rayleigh-Ritz method