

Math 257/316, Midterm 2, Section 101

17 November 2008

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned}u_t &= u_{xx} + e^{-t} \left(\sin \left(\frac{3\pi x}{2} \right) - x \right), \quad 0 < x < 1, \quad t > 0 \\u(0, t) &= 0 \text{ and } u_x(1, t) = e^{-t} \\u(x, 0) &= \sin \left(\frac{\pi x}{2} \right) + x\end{aligned}$$

[60 marks]

2. Determine the solution $u(x, y)$ to Laplace's equation in the semi-infinite strip

$$\begin{aligned}u_{xx} + u_{yy} &= 0, \quad 0 < x < 1, \quad y > 0 \\u(0, y) &= 0 \text{ and } u(1, y) = 1 \\u(x, 0) &= 2 + x, \quad |u(x, y)| < \infty \text{ as } y \rightarrow \infty\end{aligned}$$

[40 marks]