Math 257/316, Midterm 2, Section 103

November 20, 2009

Instructions. The duration of the exam is 55 minutes. Answer all questions. Calculators are not allowed.

Maximum score 100.

1. Solve the following inhomogeneous initial boundary value problem:

$$\begin{aligned} u_t &= u_{xx} - u, \qquad 0 < x < 1, \qquad t > 0 \\ u_x(0,t) &= 0, \qquad u(1,t) = \cosh(1) \\ u(x,0) &= \cosh(x) + \frac{\pi x}{2} \end{aligned}$$

Hint: You might find it helpful to know that

$$\int_0^1 x \cos\left(\left(n + \tfrac{1}{2}\right)\pi x\right)\, dx = \frac{(-1)^n}{(n+\frac{1}{2})\pi} - \frac{1}{(n+\frac{1}{2})^2\pi^2}$$

[50 marks]

2. Solve the following inhomogeneous initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= \alpha^2 u_{xx}, \qquad 0 < x < \tfrac{\pi}{2}, \qquad t > 0 \\ u_x(0,t) &= 0, \qquad u_x(\tfrac{\pi}{2},t) = 2t \\ u(x,0) &= 5 \end{aligned}$$

by using an appropriate eigenfunction expansion.

Hint: You might find it helpful to know that

$$\int_0^{\pi/2} x^2 \cos(2nx)\, dx = \frac{\pi(-1)^n}{4n^2}$$

[50 marks]