Math 217 (101): Practice Mid-Term Test 2

90 minutes

Please show all your work and justify all your responses.

1. For the iterated integral
$$\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3) dy dx,$$
sketch the region in the xy-plane that is being integrated over. Evaluate the integral.

2. Show that every plane that is tangent to the cone $x^2 + y^2 = z^2$ passes through the origin.

3. Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 1$ and the paraboloid $z = 1 - x^2 - y^2$.

4. Find the maximum and minimum values of $f(x, y) = e^{-xy}$ in the region $x^2 + 4y^2 \leq 1$, and the points at which they are attained.

5. Find the volume of the region enclosed by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ (here a, b, and c are positive constants).

6. The temperature (in degrees) in the plane is a function, $T(x, y)$, of the position (x, y). As you move through the origin with speed 5 km/h in the direction of the point $(3, 4)$, you feel the temperature *increasing* at a rate of 7 deg/h. As you move through the origin with speed $2\sqrt{2}$ km/h in the direction of the point $(-1, 1)$, you feel the temperature *decreasing* at a rate of 14 deg/h. Find $\nabla T(0, 0)$.

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