Math 217 (101): Mid-Term Test 2 **Name:**

Nov. 3, 2004: 11:00-12:30 **Student #:**

Please show all your work and justify all your responses.
You may bring (and use) a one-page (standard size, double-sided) formula sheet.
No calculators.

1.(9 pts.) Suppose the function $T(x, y, z)$ describes the temperature at a point (x, y, z) in space, with $T(1, 1, 1) = 10$, and $\nabla T(1, 1, 1) = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Suppose also that the position at time t of a particle moving through space is $(\sqrt{1+t}, \cos(t), e^t)$.

a. Compute the directional derivative of T at $(1, 1, 1)$, in the direction of the vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

b. At $(1, 1, 1)$, in what direction does the temperature decrease most rapidly?

c. Compute the rate of change of temperature experienced by the particle at time $t = 0$.

d. Write an equation for the tangent plane to the temperature level surface $T(x, y, z) = 10$ at $(1, 1, 1)$.

2. (7 pts.) Evaluate the iterated integral

$$\int_0^{\pi/2} \left[\int_{2y/\pi}^{1} \cos(y/x) dx \right] dy.$$

3. (10 pts.) Consider the function $f(x, y) = y^2\sqrt{x^2 + 9} - xy$ on the rectangle

$$D = \{(x, y) \mid |x| \leq 4, |y| \leq 1\}.$$

a. Find all the critical points of f in D, and classify them as local maxima, local minima, or saddle points.
b. Explain why we can conclude that f attains its absolute maximum and minimum values on D.
c. Find the absolute maximum and minimum values of f on D.

4. (8 pts.). Find the volume of the region bounded by the two paraboloids $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$.

5. (8 pts.) The radius r and height h of an aluminum can (a piece of a circular cylinder, with the ends included) are changing in time. At a particular time, the volume is changing at rate π, the radius is changing at rate 1, and $r = h = 1$. What is the rate of change of the surface area at this time?

6. (8 pts.) Find the maximum and minimum values of the product xyz among points (x, y, z) on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (here a, b, and c are positive numbers).