Math 217 (101): Practice Mid-Term Test 1

90 minutes

Please show all your work and justify all your responses.

1. Compute all the second partials (ie f_{xx}, f_{xy}, f_{yx}, and f_{yy}) of the function $f(x,y) = sin(e^x y)$. Verify that $f_{xy} = f_{yx}$, and explain why this relation had to hold.

2. The position of an object at time t is $\mathbf{r}(t) = \sqrt{2}t\hat{\mathbf{i}} + e^t\hat{\mathbf{j}} + e^{-t}\hat{\mathbf{k}}$. Find
   a The velocity of the object at time t.
   b The speed of the object at time t.
   c The total distance the object travels between $t = 0$ and $t = 1$.

3. Find the domain and range of the function $f(x,y) = \ln(x^2 + y^2/4 - 1)$. Sketch a few level curves of f. Sketch the graph of f.

4. The equation
$$x^3 + x + y^3 + y + z^3 + z = 6 \qquad (*)$$
   implicitly determines z as a function of x and y.
   a. Find $\partial z/\partial x$ and $\partial z/\partial y$ at the point $(x,y) = (1,1)$.
   b. Find an equation for the plane tangent to the surface defined by the equation (*) at the point $(1,1,1)$.

5. Suppose a vector function $\mathbf{r}(t)$ (with values in $\mathbf{R}^3$) is twice differentiable, and satisfies $\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0$ for all t.
   a. Show that $\mathbf{r}''(t) \cdot \mathbf{r}(t) \leq 0$.
   b. Show that the curve described by $\mathbf{r}(t)$ lies on a sphere.

6. Consider the function
$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$
   a. Show that f is continuous for all $(x,y) \in \mathbf{R}^2$.
   b. Find f_x and f_y at $(0,0)$.
   c. Show that f is not differentiable at $(0,0)$.

Sep. 29, 2004