

Math 217 (101): Mid-Term Test 1

Oct. 6, 2004: 90 minutes

Name:

Student #:

Instructions: No calculators allowed. Please show all your work.

1.(10 pts.)

- a. Find the projection of $\mathbf{v} = \langle 1, 2, 3 \rangle$ onto $\mathbf{w} = \langle 3, 2, 1 \rangle$.
- b. Find the area of the triangle with vertices $(0, 0, 0)$, $(1, 2, 3)$, and $(3, 2, 1)$.
- c. Find a parametric equation describing the line passing through the points $(1, 2, 3)$ and $(3, 2, 1)$.
- d. Find an equation for the plane containing the points $(0, 0, 0)$, $(1, 2, 3)$, and $(3, 2, 1)$.

2.(10 pts.) Consider the function

$$f(x, y) = \sqrt{x^2 + y^2 - 1}$$

- a. Find and sketch the domain of f .
- b. Find the range of f .
- c. Sketch a few level curves of f .
- d. Sketch the graph $z = f(x, y)$.

- 3.(10 pts.) Let C be the part of the curve $9y^2 = 4x^3$ in the first quadrant of the xy -plane, with $0 \leq x \leq 3$.
- Find a 2D vector function which parameterizes C (hint: one possibility is to use x as the parameter).
 - Find the length of C .
 - Suppose a particle is moving along C with speed 1. Find a vector function describing the particle's position.

- 4.(10 pts.) Consider the function $f(x, y) = xy \cos(xy^2)$.
- Compute the partial derivatives $f_x(x, y)$ and $f_y(x, y)$.
 - Use the tangent plane approximation to $f(x, y)$ at the point $(0, 10)$ to estimate $\cos(10)$.
 - In what sense is the tangent plane approximation a good one?

5.(10 pts.) Consider the function

$$f(x, y) = \begin{cases} \frac{\sqrt{|xy|}}{|x| + |y|} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}.$$

- Where in the xy -plane is f continuous?
- Compute the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ at $(0, 0)$, or else show that they do not exist.
- Is f differentiable at $(0, 0)$? Explain.