Math 217 (101): Practice Final Exam

2.5 hours

Please show all your work and justify all your responses.

1. (10 pts.) Find the flux of $\nabla \times \mathbf{F}$ upward through the hemispherical surface $x^2+y^2+z^2 = 4$, $z \geq 0$, if $\mathbf{F}$ is the vector field $\mathbf{F} = \langle z^2, x, y \rangle$.

2. (10 pts.) Let C be the curve of intersection of the parabolic cylinder $y = x^2$ and the hyperbolic paraboloid $3z = 2xy$.
(a) Write a vector parametric equation for C using x as the parameter.
(b) Find the length of the part of C between the origin and the point $(3, 9, 18)$.

3. (12 pts.) Find the maximum and minimum values of the function
$$f(x, y) = \frac{x - y}{2 + x^2 + y^2}$$
over the disk $x^2 + y^2 \leq 4$.

4. (12 pts.) Let S be the ellipsoid $x^2 + y^2 + z^2/4 = 1$.
(a) Find the volume enclosed by S.
(b) Write an expression (involving an integral) for the surface area of S, and reduce it to an integral involving only a single variable (i.e., evaluate as far as you can - you may not be able to do the final integral).

5. (12 pts.) Let S be the conical surface $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq H$, oriented with downward normal, $\mathbf{n}$, and let
$$\mathbf{F} = (x + e^{-y^2-z^2})\mathbf{i} + (x^2 + y^2 + z^2)\mathbf{j} + (z + yz + x^3)\mathbf{k}.$$
Find the flux $\int\int_S (\mathbf{F} \cdot \mathbf{n}) dS$ of $\mathbf{F}$ through S (hint: you might want to use the Divergence Theorem).

6. (10 pts.) Let S be the part of the surface $z = xy$ lying inside the cylinder $x^2 + y^2 = 3$. Find
$$\int\int_S (x^2 + y^2) dS.$$

7. (8 pts.) Suppose the equations $u = x^2 + xy - y^2$ and $v = 2xy + y^2$ define x and y implicitly as functions of u and v. Find $\frac{\partial x}{\partial u}$ at the point where $x = 2$ and $y = -1$.

8. (12 pts.) Let $\mathbf{F} = 6x^2yz^2\mathbf{i} + (2x^3z^2 + 2y - xz)\mathbf{j} + 4x^3yz\mathbf{k}$ and let $\mathbf{G} = yz\mathbf{i} + xy\mathbf{k}$.
(a) For what value of the constant λ is the vector field $\mathbf{H} = \mathbf{F} + \lambda\mathbf{G}$ conservative on $\mathbf{R}^3$?
(b) Find a scalar potential, $f(x, y, z)$ for the conservative field $\mathbf{H}$ referred to in part (a).
(c) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the curve of intersection of the two surfaces $z = x$ and $y = e^{xz}$ from the point $(0, 1, 0)$ to the point $(1, e, 1)$.