

Math 120: Assignment 9 (Due Tue. Nov. 27)

Suggested practice problems (from Adams, 6th ed.):

4.6: 1, 7, 15 **4.8:** 1, 5, 9, 13, 19, 23, 25, 31 **4.9:** 1, 5, 11, 17, 23, 25, 29

Problems to hand in:

1. Let $f(x) = x^p$ where $p = k/m$, k and m positive integers, m odd. Note that $f(0) = 0$. Apply Newton's method to f , with initial guess $x_0 \neq 0$, and find x_n . For which values of p will the sequence converge to the root 0? Fail to converge to 0? If $p = 2$ and $x_0 = 1$, how many iterations are needed to ensure an error smaller than 0.0001?
2. Let $f(x) = x^2 - 1$. Note $f(\pm 1) = 0$.
 - (a) How many iterations of Newton's method with $x_0 = 2$ are needed to get within 0.0001 of the root 1?
 - (b) Why do you think the answer is so different than in the last part of question 1?
 - (c) (**Bonus**) Determine for which values of x_0 Newton's method converges for f .
3. Find the Taylor polynomial of order n around $x = a$ for the function $f(x)$, where
 - (a) $f(x) = x^{-2/3}$, $a = 27$, $n = 3$,
 - (b) $f(x) = e^{x^3} \cos(x)$, $a = 0$, $n = 13$,
 - (c) $f(x) = \frac{1}{1+x^5}$, $a = 0$, any n
4. Approximate $(26)^{1/3}$ using the second-order Taylor polynomial for $x^{1/3}$ around $x = 27$. Estimate the error, and write the smallest interval you can be sure contains the value.
5. An electric dipole consists of two electric charges of equal magnitude and opposite sign. Suppose the charges $+q$ and $-q$ are distance d apart on the x axis. Then the electric field a distance D along the x -axis from the nearest charge is

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2}.$$

Write $E = \frac{q}{D^2} f(d/D)$ for some function f , and find the Taylor polynomial of order 2 for f around 0. How (roughly) does the electric field behave far from the dipole?

6. Find the limit
 - (a) $\lim_{x \rightarrow 0} \frac{\sin(x) - x \cos(x)}{x^3}$
 - (b) $\lim_{x \rightarrow \infty} x(\ln(x+5) - \ln(x))$
 - (c) $\lim_{x \rightarrow 0} \frac{e^x - 2 - x + \cos(x) - x^3/6}{x^4}$
 - (d) $\lim_{x \rightarrow 0^+} x^{\sin(x)}$
 - (e) $\lim_{x \rightarrow 0^+} (-\ln(x))^x$

7. Let

$$f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

- (a) Show that f is differentiable at $x = 0$, and find the linear approximation to f at $x = 0$.
- (b) Show that all derivatives of f exist at $x = 0$, and find the n -th order Taylor polynomial for f at $x = 0$. For any n , is it a "good" approximation to the function f ?