Math 120: Assignment 8 (Due Tue., Nov. 13 at the start of class)

Suggested practice problems (from Adams, 6th ed.):

4.4: 1, 3, 5, 11, 13, 15, 27, 29, 35
4.5: 1, 7, 13, 19, 21, 25, 29, 31, 38, 47
4.7: 1, 3, 5, 9, 13, 17, 21, 25, 27, 31

Problems to hand in:

1. For the following functions, determine the regions of increase and decrease, the concavity, the local and absolute extreme values, and any asymptotes, and sketch the graph $y = f(x)$
   (a) $f(x) = \frac{2x^2}{x^2-1}$
   (b) $f(x) = 2\cos(x) + \sin(2x)$
   (c) $f(x) = \frac{x^3}{x^2+1}$
2. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.
3. A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible (a) volume, and (b) surface area, of such a cylinder.
4. Use a suitable linear approximation to estimate
   (a) $\frac{1}{9.9}$
   (b) $\cos(30.5°)$
5. Suppose $f(1) = f'(1) = 1$, and $x \leq f''(x) \leq 2 - x^2$ for $1/2 \leq x \leq 1$. Find the smallest interval you can be sure contains $f(1/2)$.
6. You can run r times as fast as you can swim. You are standing on the edge of a circular swimming pool (radius R) and you wish to get to the diametrically opposite point as quickly as possible (see Example 5 Section 4.5 in text).
   (a) If $r = 2$, what is your optimal strategy?
   (b) If $r = 1$, what is your optimal strategy?
   (c) Is there any value of r for which your optimal strategy involves both running and swimming?
7. Suppose $1 < p < \infty$, and set $p' := p/(p-1)$ (so that $1/p + 1/p' = 1$).
   (a) If a and b are non-negative numbers satisfying $ab = 1$, find the smallest possible value of $\frac{a^p}{p} + \frac{b^{p'}}{p'}$.
   (b) Now if a and b are any non-negative numbers, consider $\tilde{a} := (ab)^{-1/p}a$ and $\tilde{b} := (ab)^{-1/p'}b$, and use your previous answer to prove that

$$ab \leq \frac{a^p}{p} + \frac{b^{p'}}{p'}.$$