Math 120: Assignment 7 (Due Tue., Nov. 6 at the start of class)

Suggested practice problems (from Adams, 6th ed.):

3.5: 1, 3, 9, 11, 15, 19, 23, 27, 35, 37, 39, 45, 53
2.9: 5, 7, 11, 13, 17, 19, 27
4.1: 1, 7, 13, 19, 25, 29, 33, 37
4.2: 5, 7, 11, 17, 19, 25, 31, 35, 41, 45
4.3: 5, 9, 15, 21, 25, 31, 39

Problems to hand in:

1. Simplify
   (a) $\tan^{-1}(\tan(4\pi/3))$
   (b) $\tan(\tan^{-1}(4\pi/3))$
   (c) $\tan(\sin^{-1}(x))$

2. Compute the derivative of each function, and state the domain of the function and the derivative:
   (a) $f(x) = \cos^{-1}(\sin^{-1}(x))$
   (b) $g(t) = 2^{\arctan(t)}$

3. (if you didn't do it last time) Find an equation for the tangent to the curve $xye^{x/y} + e^{-1} = 0$ at $(1, -1)$.

4. Show that the ellipse $x^2/3 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$ intersect at right angles.

5. n resistances $R_1, R_2, \ldots, R_n$ in parallel give an effective resistance R satisfying
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}.$$
   The resistances are all changing as a function of time. What condition on the rates of change ensures that at a moment when all the resistances are equal, the effective resistance R is not changing?

6. A pendulum has length 2 m.. At a moment when the pendulum makes an angle $\pi/4$ with the vertical, that angle is changing at a rate of 1/2 rad./s. How fast is the height above ground of the (end of the) pendulum changing at this moment?

7. A cylindrical tank has (horizontal) cross-sectional area $10m^2$. It leaks water at a rate of $20m^3/min$. At what rate does it need to be filled in order that the height of the water increase at a rate of $2m/min$?

8. Find the local and absolute extreme values of $f(x) = x^{2/3} - x^{8/3}$ on $[-2, 2]$ and the points where they are attained.

9. Consider the function $f(x) := \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$.
   (a) Show f is continuous on $[0, \infty)$ but has neither a local max. nor a local min. at the endpoint $x = 0$.
   (b) Show f is differentiable in $(-\infty, \infty)$, and has a critical point at $x = 0$ which is neither a local max., local min., nor inflection point.