Assignment # 1 Solutions

1) a) $x^3 + x^2 \le 2x \iff x^3 + x^2 - 2x \le 0$

$\iff x(x^2 + x - 2) \le 0$

$\iff x(x+2)(x-1) \le 0$

$\iff x \in \boxed{(-\infty, -2] \cup [0, 1]}$

b) $|3 - 1/x| < 2 \iff -2 < 3 - 1/x < 2$

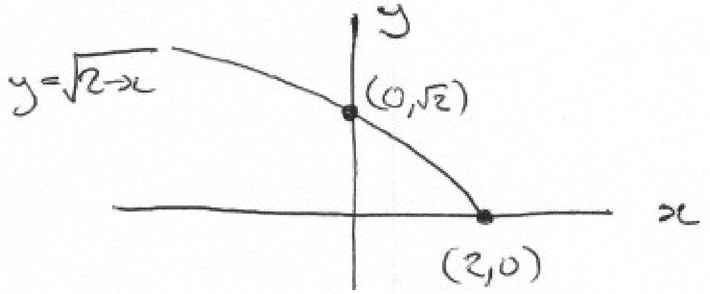
$\iff -5 < -1/x < -1$

$\iff 5 > 1/x > 1$

$\iff 1/5 < x < 1$

$\iff x \in \boxed{(1/5, 1)}$

2) $f(x) = \sqrt{2-x}$. Domain: $2 - x \ge 0 \iff \boxed{x \le 2}$



3) The points $(a,0)$ and $(0,b)$ determine a straight line (or infinitely many if $a=b=0$) through the origin, which is a graph $y = mx$ for some m, unless it is vertical. So the only restriction is: $\boxed{a \ne 0, \text{ unless } b = 0}$.

4) a) for $t \neq 0$, $\frac{t^2-3t^3}{t+3t^3} = \frac{t-3t^2}{1+3t^2}$. As $t \to 0$,

the numerator $\to 0$, while the denominator $\to 1$, hence

$$\lim_{t \to 0} \frac{t^2-3t^3}{t+3t^3} = \frac{0}{1} = \boxed{0}.$$

b) for $x \neq 9$, $\frac{3-\sqrt{x}}{x-9} = \frac{3-\sqrt{x}}{(\sqrt{x}-3)(\sqrt{x}+3)} = \frac{-1}{\sqrt{x}+3}$,

so $$\lim_{x \to 9} \frac{3-\sqrt{x}}{x-9} = \lim_{x \to 9} \frac{-1}{\sqrt{x}+3} = \frac{-1}{3+3} = \boxed{-1/6}$$

c) $\sqrt{x-1}$ is not defined for $x<1$, so

$\lim_{x \to 0} \sqrt{x-1}$ $\boxed{\text{does not exist}}$ ($0<|x|<\delta$ does <u>not</u> imply x is in the domain of $\sqrt{x-1}$ for <u>any</u> $\delta>0$).

d) $$\frac{\sqrt{t^2}}{t} = \frac{|t|}{t} = \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$$

So $\lim_{t \to 0+} \frac{\sqrt{t^2}}{t} = 1 \neq -1 = \lim_{t \to 0-} \frac{\sqrt{t^2}}{t}$ and $\lim_{t \to 0} \frac{\sqrt{t^2}}{t}$ $\boxed{\text{does not exist}}$.

5) a) $f(x) \equiv 0$ is both even and odd.

b) $$f(x) = \underbrace{\left[\frac{f(x)+f(-x)}{2}\right]}_{\text{even}} + \underbrace{\left[\frac{f(x)-f(-x)}{2}\right]}_{\text{odd}}.$$