Question: Is $L$ invertible?

Answer: Yes! In fact,

$\det (L) = \pm N$

Exercise: check for $N=4$.

What remains? bringing $q(x)$.

\[\begin{align*}
\frac{d^2 u}{dx^2} + q(x) u(x) &= r(x) \\
\mu(0) &= A \\
\mu(1) &= B
\end{align*}\]

(solve on $[0,1]$)

We need to discretize $q(x)\mu(x)$ on $x_1, \ldots, x_{N-1}$

\[\begin{bmatrix}
q(x_1) u(x_1) \\
q(x_2) u(x_2) \\
q(x_{N-1}) u(x_{N-1})
\end{bmatrix} = QU
\]

If we write $y_j = y_{j+1} - q_j y_j$, $q_j = 9_j$

\[Q = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
q_1 & 0 & \cdots & 0 \\
0 & q_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & q_{N-1}
\end{bmatrix}
\]

\[y_0 = f_0, \quad y_N = f_N
\]

Then our BVP is approximated by:

\[(L + \Delta x^2 Q) U = \Delta x^2 R
\]

BC (left)

BC (right)

What do we need?
- A set of vectors $V (\mathbb{R}^n)$
- Vector addition in $V$
- A set of scalars $F.$
  ($V = \mathbb{R}^n, F = \mathbb{R}$), with multiplication.

What is a vector?
- An $n$-tuple is a vector:
  $$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$
- $P(x) = ax^3 + bx^2 + cx$
  $P(x) \in \mathbb{R}[x]$.
- $f(x) = \sin(x)$
  $f(x) \in C^\infty(\mathbb{R})$. 