Problem Set 6. Part I. Due on Tuesday, October 27, but it's better if you try it before te midterm. Part II will be assigned right after the midterm.

(1) Prove that if G, H are groups, then the set $G \times H$ with the operation defined by

$$(g_1, h_1) \circ (g_2, h_2) = (g_1 g_2, h_1 h_2)$$

is a group. (We have been assuming this all along when talking about the product of groups).

(2) Let G, H be groups, with the idenitity elements e_G and e_H, respecively. Let $\tilde{G}$ be the subgroup of $G \times H$ defined by

$$\tilde{G} = \{(g, e_H) \mid g \in G\}$$
$$\tilde{H} = \{(e_G, h) \mid h \in H\}.$$

Prove that $\tilde{G}$ and $\tilde{H}$ are normal subgroups of $G \times H$.

Prove that $\tilde{G}$ is isomorphic to G, and $\tilde{H}$ is isomorphic to H.

(3) Let K be the direct product of G and H: $K = G \times H$. Prove that $K/\tilde{G} \simeq H$, and $K/\tilde{H} \simeq G$.

(4) Give an example of a group G and a normal subgroup N, such that G is *not* isomorphic to $N \times (G/N)$. Can you make such an example with an *abelian* group G?