Problem Set 5, Due Thursday October 15.

1-4. Remember to do Problems 30, 31, 32, 33 from the book.

Definition. Let G be a group. The *centre* of G is the set $Z(G) \subset G$ that consists of all elements that commute with every element of G:

$$Z(G) = \{h \in G \mid hg = gh \text{ for all } g \in G\}.$$

Problem 5.

(a) Show that $Z(G)$ is a subgroup of G.
(b) Give an example of a (nontrivial) group G such $Z(G) = G$. Also, give an example of a nontrivial group G such that $Z(G) = \{e\}$.
(c) Prove that $Z(G)$ is a normal subgroup in G.

Problem 6. Let $f : G \to H$ be a group homomorphism. Prove that for every $x \in G$, $f(x^{-1}) = (f(x))^{-1}$.

Problem 7. Let H be a subgroup of G that satisfies: $gHg^{-1} \subset H$ for every $g \in G$. Prove that H is normal.