Problem Set 11th and the last. Due Thursday December 3.

(1) Prove that $\mathbb{Q}[x]/(x^2 - 2x - 1)$ is a ring isomorphic to $\mathbb{Q}[\sqrt{2}]$. (*Hint*: Use a root of $x^2 - 2x - 1$ to construct a homomorphism from $\mathbb{Q}[x]$ to $\mathbb{Q}[\sqrt{2}]$, and then apply the first isomorphism theorem).

(2) Let $\omega \in \mathbb{C}$ be a root of the polynomial $x^5 - 3x + 6$. Prove that the subring R of $\mathbb{C}$ defined by

$$R = \{a_0 + a_1\omega + \cdots + a_4\omega^4 \mid a_i \in \mathbb{Q}, i = 0, \ldots, 4\}$$

is a field (you do not need to prove that R is a ring).

(3) (from last year's final exam). Factor the following elements of a ring R into irreducible elements of R. Explain why the factors you give are irreducible.
   (a) 15, an element of $R = \{a + b\sqrt{-2} \mid a, b \in \mathbb{Z}\}$.
   (b) $x^3 + 1$, an element of $\mathbb{F}_7[x]$.

(4) Section 4.10: Problem 26.
(5) Section 3.6: Problem 31.
(6) Section 3.6: problem 33.