Math 600: Modular forms

Instructor: Julia Gordon
Office: Math 217
e-mail: `gor at math.ubc.ca`

Text: There is no required text. Main sources (all available online):

- James Milne's notes.
- T. Miyake, "Modular forms" (available online at UBC library).
- William Stein, "Modular Forms, a computational approach"
- S. Lang, "Introduction to modular forms" and "Elliptic Functions".
- F. Diamond and J. Shurman "Introduction to modular forms" (available online at UBC library).
- Zagier's lectures in "1-2-3 of modular forms" by J. H. Bruinier, G. van der Geer, G. Harder, and D. Zagier.

Course outline: Modular forms is a classical subject with roots in complex analysis but central in Number theory, and with connections to nearly all of mathematics from Algebraic geometry to Coding theory. The goal of this course is to cover the classical theory of modular forms and emphasize as many connections, applications, and points of view as possible (in later parts, at the cost of some details of proofs). In particular, the topics will include:

I. The basic definitions and examples:

- Riemann surfaces; the upper half-plane; doubly-periodic functions. The modular curve.
- Modular forms for $SL_2(\mathbb{Z})$ and principal congruence subgroups.
- Eisenstein series
- theta-series and sums of squares.
- Cusp forms; Ramanujan's Delta.

II. The space of modular forms:

- Dimension formulas.
- Hecke operators; Petersson inner product; the basis of eigenforms.
- Newforms and oldforms.
- Modular symbols and explicit computation.

III. L-functions and Fourier expansions.

- L-functions attached to modular forms
- Connection with elliptic curves and Galois representations
- Ramanujan conjecture

If possible and depending on the audience's background and interests, we will also cover some of the following topics:

- Eichler-Shimura theory
- applications in Number theory (e.g. transcendence results, explicit class number computations)
- adeles, representation theory, Automorphic forms perspective
- Siegel modular forms

Background expectations. The course is very beginner-friendly; the only prerequisite is some knowledge of complex analysis; basic Number theory would help. Some Algebraic geometry and representation theory is helpful but is not required.

Marking. The mark will be based on homework and a final talk (I hope everyone would give one talk on one topic of interest towards the end of the course).