Growing list of problems for Math534.

All Lie algebras are assumed to be finite-dimensional and over \(\mathbb{C}\), unless otherwise specified. The references are: [H] – Humphreys; [FH] – Fulton & Harris.

The exercises not marked with a check mark are to be discussed during the next problem session in class.

1. (a) Show that the Lie algebras \(\mathfrak{sl}_n\), \(\mathfrak{so}_n\) (with \(n > 2\)), \(\mathfrak{sp}_n\) have the property \([\mathfrak{g}, \mathfrak{g}] = \mathfrak{g}\). (this is Exercise 9 on p. 5 of [H]).

   (b) Show that the derived algebra of \(\mathfrak{gl}_n\) is \(\mathfrak{sl}_n\) (this is [H], Exercise 2 on p.9)

2. (a) Humphreys, Exercise 7 on p.5.

   (b) Humphreys, Exercise 3 on p.10.

3. Show that the smallest dimension of a faithful representation of the Heisenberg Lie algebra (defined in the written homework) is 3.