Growing list of problems for Math534.

All Lie algebras are assumed to be finite-dimensional and over \( \mathbb{C} \), unless otherwise specified. The references are: [H] – Humphreys; [FH] – Fulton & Harris.

The exercises not marked with a check mark are to be discussed during the next problem session in class.

1. (a) Show that the Lie algebras \( \mathfrak{sl}_n, \mathfrak{so}_n \) (with \( n > 2 \)), \( \mathfrak{sp}_n \) have the property \([g, g] = g\). (this is Exercise 9 on p. 5 of [H]).
   
   (b) Show that the derived algebra of \( \mathfrak{gl}_n \) is \( \mathfrak{sl}_n \) (this is [H], Exercise 2 on p.9)

2. (a) Humphreys, Exercise 7 on p.5.
   
   (b) Humphreys, Exercise 3 on p.10.

3. Show that the smallest dimension of a faithful representation of the Heisenberg Lie algebra (defined in the written homework) is 3.

4. Humphreys, Exercise 3 on p.20

5. Humphreys, Exercise 4 on p.20