Math 534. Written problems, set 4. Due December 13.

(1) Let V be the standard (3-dimensional) representation of $\mathfrak{sl}_3(\mathbb{C})$.
   (a) Draw the weight diagram of V.
   (b) Draw the weight diagram for $\operatorname{Sym}^2 V$ (with multiplicities).
   (c) Prove that the representation $\operatorname{Sym}^n V$ is irreducible for all n.

(2) This problem is about representations of $\mathfrak{sp}_4(\mathbb{C})$.
   (a) Draw the root lattice and the weight lattice for C_2 on the same diagram.
   (b) Draw the weight diagram for the irreducible representation (call it W) of $\mathfrak{sp}_4(\mathbb{C})$ with highest weight $\alpha+\beta$ (where $\{\alpha, \beta\}$ is the standard base for C_2).
   (c) Let V be the standard (4-dimensional) representation of $\mathfrak{sp}_4$. Show that the alternating square $\wedge^2 V$ is the direct sum of W from part (b), and the trivial representation.
   (d) Show that $\operatorname{Sym}^2 V$ is isomorphic to the adjoint representation.