Math 534. Written problems, set 2. Due Tuesday, October 11.

1) Let $\mathfrak{g}$ be a Lie algebra, and let $V$ and $W$ be $\mathfrak{g}$-modules. Let $\text{Hom}(V, W)$ be the space of all linear operators from $V$ to $W$. Define the action of $\mathfrak{g}$ on $\text{Hom}(V, W)$ by:

$$ (X \cdot f)(v) = X \cdot f(v) - f(X \cdot v), \quad X \in \mathfrak{g}, f \in \text{Hom}(V, W). $$

(a) Show that this makes $\text{Hom}(V, W)$ an $\mathfrak{g}$-module.

(b) Show that $\text{Hom}(V, W)$ is isomorphic to $V^* \otimes W$.

2) Humphreys, Exercise 6 on p.24.

3) Humphreys, Exercise 2 on p.34.