
1. Find an example of an irreducible algebraic group such that the set of its real points is not connected.

2. Prove that there are no nontrivial homomorphisms from a quasitorus to the additive group $G_a$ ($G_a$ is the notation for the algebraic group s.t. $G_a(K) = K$).

3. Show that the maps exp and log between the groups of nilpotent (respectively, unipotent) operators are morphisms that are inverses of each other. (Hint: work with the formal power series).

4. Let $G$ be an algebraic group, let $g \in G(\mathbb{C})$. Show that if $g^m$ is semisimple for some positive integer $m$, then $g$ is semisimple.

5. Show that the intersection of the kernels of all characters of an algebraic group $G$ is a normal algebraic subgroup, and the quotient of $G$ by this subgroup is a quasitorus.

6. Let $T_1, T_2$ be algebraic tori. Then there is one-to-one correspondence between $\text{Hom}(T_1, T_2)$ and $\text{Hom}(X(T_2), X(T_1))$.

7. Show that any subgroup of an irreducible solvable algebraic group $G$ that consists only of semisimple elements has to be commutative. In particular, any finite subgroup of such $G$ is commutative.