
1. Let \( g \) be a Lie algebra that is a semidirect sum of \( g_1 \) and \( g_2 \) (that is, as a vector space it is a direct sum, and \( g_1 \) is an ideal). Let us define the map \( \beta : g_2 \rightarrow gl(g_1) \) by the formula \([X_2, X_1]_g = \beta(X_2)X_1\). Show that for every \( X_2 \in g_2, \beta(X_2) \) is a derivation of \( g_1 \).

2. (a) Show that \((GL_n(K))' = SL_n(K)\).
   (b) Show that \(O_n(K)' = SO_n(K)\).
   (c) Show that \(U_n' = SU_n\).

3. Show that \(SL_n(K)\) is semisimple.

4. Show that the radical of a complex Lie group coincides with the radical of the same group if you considered it as a real Lie group.

5. Show that any nontrivial connected solvable Lie group has a connected normal subgroup of codimension 1.

6. Show that weight subspaces corresponding to different weights are linearly independent.

7. Let \( H \) be a normal subgroup of \( G \), and let \( \chi \) be a character of \( H \), and \( R \) — a representation of \( G \). Prove that for any \( g \in G, R(g)V_\chi(H) = V_{\chi^g}(H) \), where \( \chi^g \) is another character of \( H \) defined by \( \chi^g(h) = \chi(g^{-1}hg) \).