Math 323: Homework 7

1. Section 13.1 Problem 1. Show that $p(x) = x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$. Let θ be a root of $p(x)$. Find the inverse of $1 + \theta$ in $\mathbb{Q}(\theta)$.

2. Section 13.1 Problem 3. Show that $x^3 + x + 1$ is irreducible over $\mathbb{F}_2$ and let θ be a root. Compute the powers of θ in $\mathbb{F}_2(\theta)$.

3. Section 9.4 Problem 1. (b,c,d) Determine whether the following polynomials are irreducible in the rings indicated. For those that are reducible, determine their factorization into irreducibles. The notation denotes the finite field $\mathbb{Z}/p\mathbb{Z}$, where p is a prime.

(a) $x^3 + x + 1$ in $\mathbb{F}_3[x]$
(b) $x^4 + 1$ in $\mathbb{F}_5[x]$
(c) $x^4 + 10x^2 + 1$ in $\mathbb{Z}[x]$

4. Prove that the following polynomials are irreducible:

(a) $x^9 + 5x^7 - 15x^6 + 30x - 20$ in $\mathbb{Q}[x]$.
(b) $x^6 - y(y-1)x^4 + (y-1)x^3 - (y^2 - 2y + 1)x + y - 1$ in $\mathbb{R}[x, y]$.
(c) $y^3 - x^2y + xy^2 - 3x$ in $\mathbb{F}_5[x, y]$.

5. Section 9.5 Problem 3. Let p be an odd prime in $\mathbb{Z}$ and let n be a positive integer. Prove that $x^n - p$ is irreducible over $\mathbb{Z}[i]$.