*This midterm has **5 questions** on **6 pages**, for a total of 50 points.*

Duration: 50 minutes

- **you need to justify all your answers.**
- Continue on the back of the previous page if you run out of space.
- One page formula sheet is allowed. No other documents, and no electronic devices of any kind (including calculators, cell phones, etc.) are allowed.
- Unless a problem states otherwise, you **do not** have to simplify algebraic expressions to the shortest possible form, and do not have to evaluate long numerical expressions.

Full Name (including all middle names): ______________________

Student-No: ______________________

Signature: ______________________

Question:	1	2	3	4	5	Total
Points:	15	13	12	6	4	50
Score:						

$\boxed{\text{2 marks}}$ 1. (a) Find a unit vector that has the same direction as $\langle 1, 3, -1 \rangle$.

$\boxed{\text{4 marks}}$ (b) Find the distance from the point $(0, 0, 1)$ to the plane with the equation $x+3y-z = 1$.

$\boxed{\text{5 marks}}$ (c) Find an equation of a plane parallel to the plane with the equation $x + 3y - z = 1$ and at the distance 5 from it. How many such planes are there?

$\boxed{\text{4 marks}}$ (d) Find the angle between the edge and the diagonal of a 'unit cube' in $\mathbb{R}^5$ (that is, the 'box' spanned by the unit vectors of the five axes). (You can use inverse trig functions in the answer).

2. *(In this problem, the parts are related. If you could not do an earlier part, you can give some notation to the answer for it, and proceed to the later parts using that notation, to get partial credit).*

We make the convention that he x-axis points East, the y-axis points North, and the z-axis points up. The unit on each axis is 1km. Suppose a small airplane is moving along a straight line L_1. At $t = 0$, it is at the point $(0, 0, 3)$ (that is, its altitude is 3km). The projection of its trajectory onto the xy-plane points directly North-East, its altitude is increasing at 100m/min, and its ground speed (i.e. the speed at which its projection to the ground is moving) is 10km/min.

3 marks (a) Find the velocity and speed of the airplane (velocity is a 3d-vector, and speed is the magnitude of the velocity).

3 marks (b) Find the parametric equations for the line L_1 (the trajectory of the airplane).

2 marks (c) Suppose another airplane is flying along the line L_2 with the parametric equation $\mathbf{r}(t) = \langle 0, 10t, 2\sqrt{2} \rangle$. Find the symmetric equations for L_2.

5 marks (d) Does there exist a plane that contains the trajectories of the both airplanes? If yes, give its equation.

3. Consider the function $f(x, y) = \ln\left(a + \frac{y}{x^2}\right)$, where a is a parameter.

$\boxed{\text{4 marks}}$ (a) Determine the domain and range of the function f (consider the cases $a = 0$, $a > 0$, and $a < 0$ separately).

$\boxed{\text{4 marks}}$ (b) Suppose $a = -1$. Is the domain of f a closed set, an open set, or neither? (Include brief proof using the definition of an open/closed set).

$\boxed{\text{4 marks}}$ (c) What do the level curves $f(x, y)$ look like in each of the cases? Describe and sketch them.

4. As usual, let ∂S denote the boundary of a set S. Recall that the intersection of two sets A and B is the set of their common elements: $A \cap B = \{x | x \in A \text{ and } x \in B\}$, and the union of the two sets A and B is the set of elements that belong to at least one of them: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

$\boxed{\text{3 marks}}$ (a) Prove that $\partial(S_1 \cup S_2) \subseteq \partial S_1 \cup \partial S_2$ (that is, for any two sets S_1 and S_2 in $\mathbb{R}^n$, the boundary of their union is contained in the union of their boundaries).

$\boxed{\text{3 marks}}$ (b) Is it true or false that $\partial(S_1 \cap S_2) \subseteq \partial S_1 \cap \partial S_2$? (Prove or give a counterexample).

$\boxed{\text{4 marks}}$ 5. Find the equation of the surface S consisting of all points in $\mathbb{R}^3$ that are equidistant from the point $(0, 0, 0)$ and the plane $z = 4$. If you know the name of this surface, name it and sketch it.