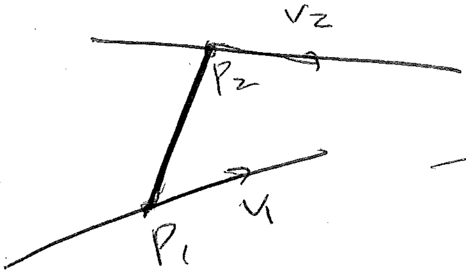
Assume : not parallel.



$$\boxed{\overline{P_1P_2} \cdot (\bar{v}_1 \times \bar{v}_2) = 0}$$

detects whether the lines intersect

intersect $\Longleftrightarrow$

P_1P_2 lies in the plane of $\bar{v}_1$ and $\bar{v}_2$ &

$\Longleftrightarrow$ P_1P_2 $\perp$ normal to this plane which is $\bar{v}_1 \times \bar{v}_2$

This is an addition to last class' lecture: how to detect whether lines in space are skew or intersecting, without solving equations.

this is equivalent to saying that the distance between these lines is 0.

(which is the same as saying they have a common point).

Range of f :

if $z = \sqrt{4-x^2-y^2}$, then $z \geq 0$

Also $z \leq 2$, because $x^2+y^2 \geq 0$
so $4-x^2-y^2 \leq 4$

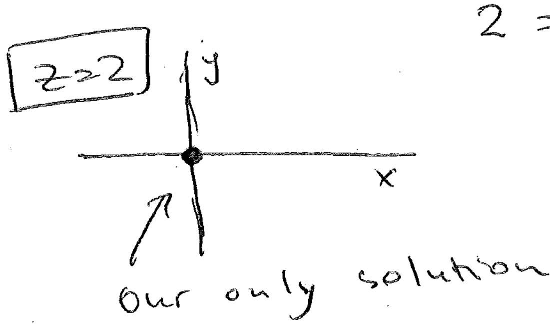
We proved:
$\text{Range}(f) \subseteq [0,2]$

Let us show $\text{Range}(f) = [0,2]$.

Need: for every $z \in [0,2]$, prove
$\exists\ (x,y) \in D$ s.t. $f(x,y) = z$.

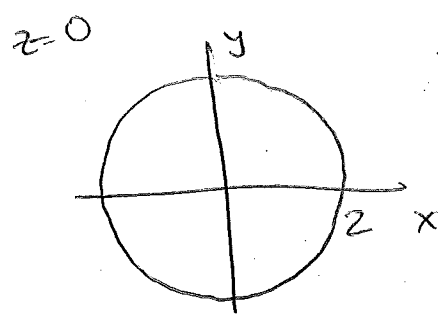
So, need to solve: $\leftarrow$ find (x,y).
$$z = \sqrt{4-x^2-y^2}$$

<u>example</u>: ① $z=2$.
$$2 = \sqrt{4-x^2-y^2} \Leftrightarrow x^2+y^2=0$$
$$\Leftrightarrow (x,y) = (0,0).$$



② $\underline{z=0}$

Get: $0 = \sqrt{4-x^2-y^2}$
$$4-x^2-y^2=0$$
$$x^2+y^2=4. \quad \text{- whole circle of radius 2.}$$



Functions of several (2 or 3) variables.

$f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$D \subset \mathbb{R}^n$, $V \subset \mathbb{R}$

D - set of points at which f is defined

D called the **domain** of f

V = the set of values of f

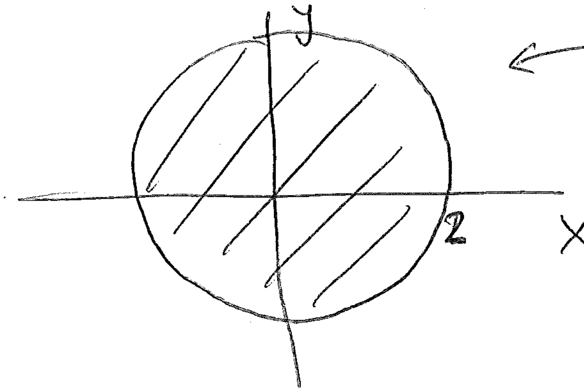
= range of f

$= \{ y \in \mathbb{R} \mid \exists (x_1, \dots, x_n) \in \cancel{\mathbb{R}^n}\ D$ s.t. $f(x_1 \dots x_n) = y \}$

Example $f(x,y) = \sqrt{4 - x^2 - y^2}$

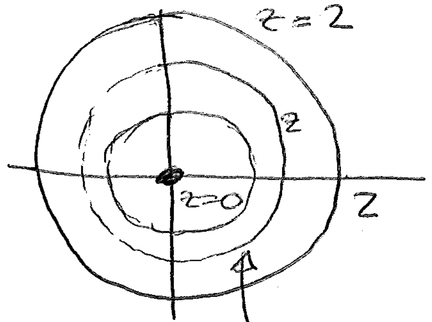
Domain of f

$= \{ (x,y) \mid x^2 + y^2 \leq 4 \}$



closed disc of radius 2 centred at $(0,0)$

③ Now when $z \in (0,2)$



$$z = \sqrt{4-x^2-y^2}$$

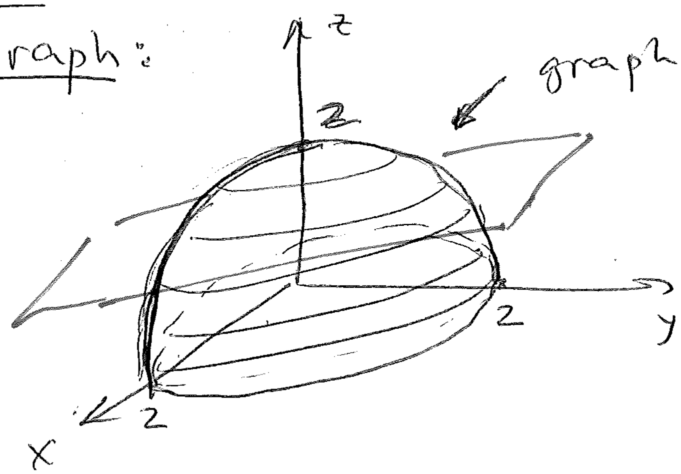
$$z^2 = 4-x^2-y^2$$

$$x^2+y^2 = 4-z^2$$

circle of radius $\sqrt{4-z^2}$ ← in particular, $z \in \mathrm{Range}(f)$

If we draw the set of solutions to $f(x,y) = z$ for equally spaced values of z in the range of f, we get "contour plot" or "level curves" of the function f.

Graph:



$z = f(x,y) = \sqrt{4-x^2-y^2}$ (it is ~~looks like~~ half a sphere).

To get level curves, we slice it with horizontal planes, and then put all these curves on one plane.

Next, we discussed the contour plot of the "mystery function" - also posted on the website.

The next two pages are the discussion of that plot.

Contourplot of a mystery function:

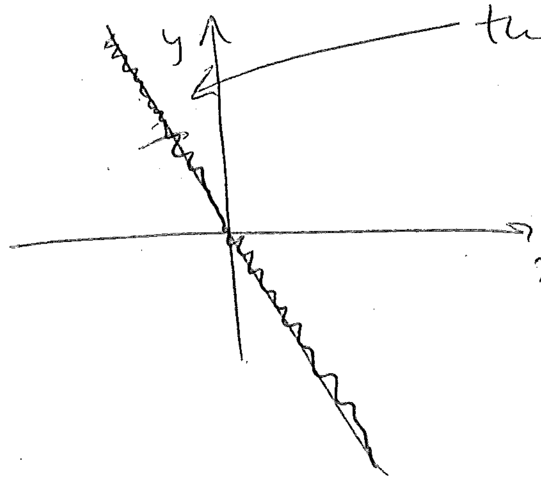
* Features: a lot of it the same colour meaning it changes very slowly there

* there is a 'discontinuity' (starts changing fast!)

* Domain: centre circle is not in the domain

  f not defined in the disc of radius 2 centred at (0,0).
  (hard to tell open or closed).

this line not in the domain?



* Can you prove that the points of intersection of the line with the circle (are not in) the domain?

$$\boxed{\text{level curves of a function cannot intersect}}$$

* Every point in the domain has a level curve passing through it.

In our pictures ~~[illegible]~~ there are no curves from the sampling (pick equally spaced z^*-values)

no curves pass through the big orange domain.

This means the function is 'almost constant' there.

Guess the formula:

$$f(x,y) = \frac{\sqrt{Px^2 + Qy^2 - R}}{ax + by + c}$$

our "bad line"

Note

$ax^2 + by^2 = c$

ellipse if $a, b > 0$

hyperbola if $a > 0$, $b < 0$ or $a < 0$, $b > 0$.