* Under Homework for this week
there is a "click here" for proofs, logic, ...
Please read, think about, come talk to me!

* HW will be posted by Thurs. morning.

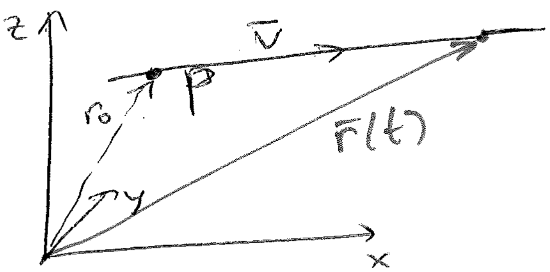
* Midterm, in class Oct. 5.
Will cover: vector, $\cdot$, $\times$, projections, lines/planes,
one proof problem.

Today: more tricks about lines/planes.

We stopped at: parametric equations of a line:

- vector form

line through P parallel to $\bar{v}$



$$\bar{r}(t) = \bar{r}_0 + t\bar{v}$$

(function $\bar{r}: \mathbb{R} \to \mathbb{R}^3$)
input: t, output: $\bar{r}(t)$ computed by this formula).

$$\bar{v} = \langle a, b, c \rangle$$

$P = (x_0, y_0, z_0)$

$$\begin{cases} x(t) = x_0 + at \\ y(t) = y_0 + bt \\ z(t) = z_0 + ct \end{cases}$$

- good for plotting a line

Not so good for detecting whether a given point (x, y, z) belongs to this line.

Outside viewpoint:

* Let's try to solve for t:

if our point (x, y, z) is on the line then there should exist t, s.t.

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \qquad (*)$$

and vice versa (if such t exists, this point lies on the line).

($(x, y, z) \in L$ if and only if $\Longleftrightarrow$ exists t satisfying the system of 3 equations above $\to (*)$).

our line

$$\begin{cases} \dfrac{x - x_0}{a} = t \\ \dfrac{y - y_0}{b} = t \\ \dfrac{z - z_0}{c} = t \end{cases}$$

– all three should hold!

$$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

– symmetric equation of the line.

(!) It is 2 equations

Two equations are expected to define a line in $\mathbb{R}^3$.

If I want to define a line in $\mathbb{R}^5$, how many equations on $(x_1, x_2, x_3, x_4, x_5)$ do we need?

– need 4 equations.

A line is an intersection of 2 planes:
symmetric equation gives two convenient planes:

$$\begin{cases} \dfrac{x-x_0}{a} = \dfrac{y-y_0}{b} & \text{– plane parallel to } z\text{-axis} \\[2ex] \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c} & \text{– plane parallel to } x\text{-axis} \end{cases}$$

(in fact there's also the third plane

$\dfrac{x-x_0}{a} = \dfrac{z-z_0}{c}$ passing through this line;

any two planes are enough to specify the line).

* Important: what if a, b or c is zero?

Then, for example if $a = 0$,
the symmetric equation is:

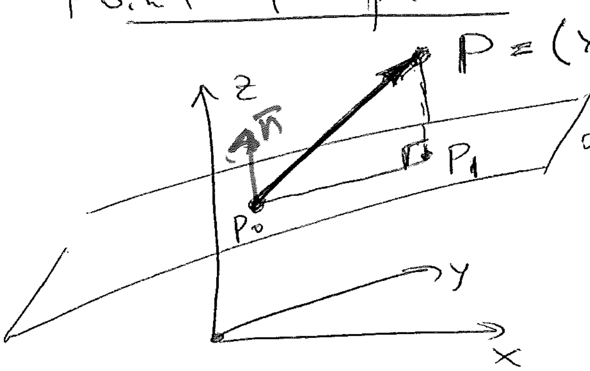
$$\boxed{\begin{cases} x = x_0 \\ \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c} \end{cases}}$$

If $a = b = 0$, get:

$$\begin{cases} x = x_0 \\ y = y_0 \end{cases}$$ – a line through (x_0, y_0) parallel to z-axis.

<u>Distances</u>:
- from point to plane
- from point to a line.

<u>Point to plane</u>



α – plane with equation
$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\boxed{\text{Distance from } P \text{ to the plane } \alpha = ?}$$

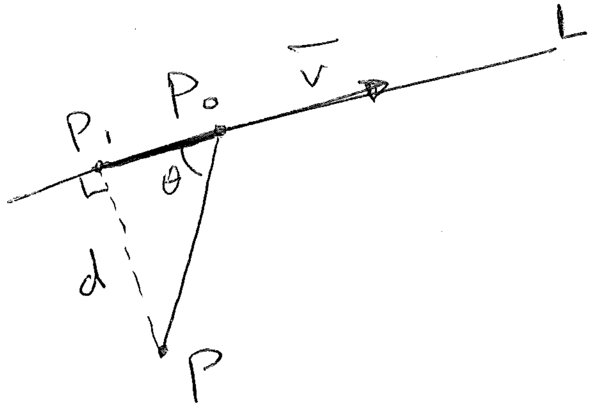
<u>Trick</u>: $d = \left| \operatorname{comp}_{\bar{n}} \overline{P_0P} \right|$

$\bar{n}$ = normal vector to α $= \langle a, b, c \rangle$

$$= \frac{\overline{P_0P} \cdot \bar{n}}{|\bar{n}|}$$

$$= \frac{|a(x_1-x_0) + b(y_1-y_0) + c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

Distance from point to a line



One way:

write an equation of a plane through P perpendicular to L ← our line

Then find P_1 = the intersection point of this plane with L.

$$d = |PP_1|.$$

- **Better way** (in $\mathbb{R}^3$)

$$d = |PP_0| \sin\theta = \frac{|\overline{P_0P} \times \bar{v}|}{|\bar{v}|}$$

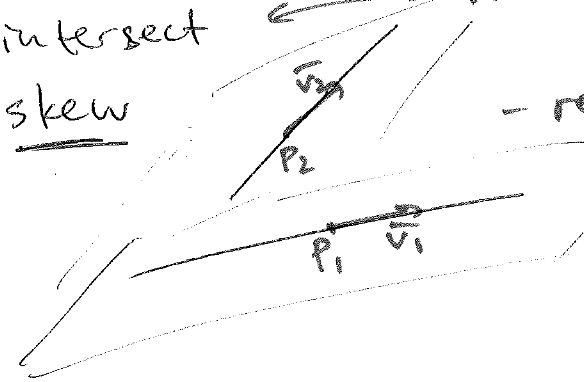
- **Distance between lines**

i) What can happen: two lines in $\mathbb{R}^3$ can be

- parallel
- intersect ← distance = 0
- **skew**

with probability 1 random two lines in space are skew

- read 10.4 (or derive)



• suppose you are given two lines.

How to tell if they are

- parallel
- intersecting
- skew ?

Example

$$\begin{cases} \bar{r}_1(t) = \underset{P_0}{\langle 1,0,3\rangle} + t\underset{\bar{v}_1}{\langle 1,1,2\rangle} \\ \bar{r}_2(t) = \underset{P_1}{\langle 3,0,1\rangle} + t\underset{\bar{v}_2}{\langle 1,2,1\rangle} \end{cases}$$

Not parallel (parallel: direction vectors should be proportional

$\langle 1,1,2\rangle \neq k\langle 1,2,1\rangle$.)

• How to ~~[illegible]~~ to check by hand if they intersect:

$$\begin{cases} 1+t = 3+\cancel{t}\,s \\ 0+t = 0+2s \\ 3+2t = 1+s \end{cases}$$

– try to solve for t, s.

If solution exists, they intersect.

Otherwise, skew.

$$\boxed{\text{Next time: } 12.1}$$

Or: use the trick:

Distance formula says, distance btw them is $\dfrac{|\overline{P_0P_1} \times (\bar{v}_1 \times \bar{v}_2)|}{|\bar{v}_1 \times \bar{v}_2|}$

This distance is 0 if and only if they intersect, ie. $\overline{P_0P_1} \perp (\bar{v}_1 \times \bar{v}_2)$.

So can check if $\langle 2,0,-2\rangle \perp \langle 1,1,2\rangle \times \langle 1,2,1\rangle$.