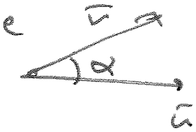
Last time:

- Wanted to define cross product so that:

$\cdot\ \bar{u}\times\bar{v} \perp \bar{u}, \bar{v}$ + right-hand rule

(1) $\cdot\ |\bar{u}\times\bar{v}| = |\bar{u}||\bar{v}|\sin\alpha$



- Defined an algebraic operation

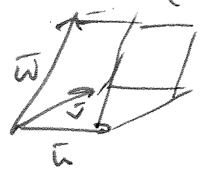
$$\bar{u}\underset{\approx}{\times}\bar{v} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \bar{u}\times\bar{v} \leftarrow \text{as in (1)}$$

want to show

defined the operation for i, j, k.
used "distribution laws"

- Know: our operation $\bar{u}\underset{\approx}{\times}\bar{v}$ has all these good properties, and satisfies $(\bar{u}\underset{\approx}{\times}\bar{v})\cdot\bar{w}$

= volume of the box



for any $\bar{w}$.
$= \langle a_0, b_0, c_0\rangle$

really proved $= \begin{vmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ fact $\leftarrow$ later this fall.

- But we also proved using the parallelograms that $(\bar{u}\times\bar{v})\cdot\bar{w}$ = volume of the box.

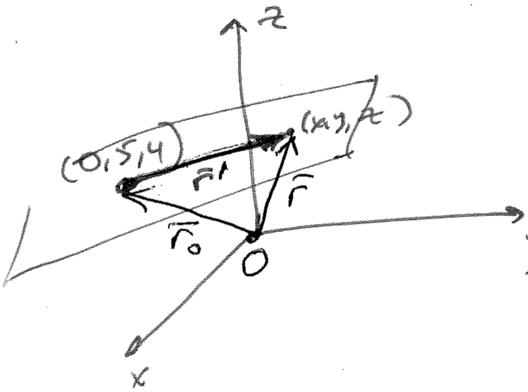
- Get: for any $\bar{w}$, $(\bar{u}\times\bar{v})\cdot\bar{w} = (\bar{u}\underset{\approx}{\times}\bar{v})\cdot\bar{w}$

Then $\bar{u}\times\bar{v} = \bar{u}\underset{\approx}{\times}\bar{v}$ $\leftarrow$ so this is the cross product.

Equations of Lines and planes in $\mathbb{R}^3$

* **How to define a plane in $\mathbb{R}^3$.**

Example: write an equation of the plane in $\mathbb{R}^3$ perpendicular to $\bar{n} = \langle 1, 2, 3 \rangle$ and passing through $(0, 5, 4)$.



The vector $\bar{r}'$ connecting a point (x, y, z) in the plane with $(0,5,4)$ lies in the plane.

Then $\bar{r}' \perp \bar{n}$.

Then $\bar{n} \cdot \bar{r}' = 0$. ← equation of a plane in vector form

$\bar{r}' = \langle x, y-5, z-4 \rangle$

(can rewrite it as: $\bar{n} \cdot (\bar{r} - \bar{r}_0) = 0$; $\bar{r}$: vector $\langle x, y, z \rangle$)

So, we get: $\bar{n} \cdot \bar{r}' = 0$

$$\langle 1, 2, 3 \rangle \cdot \langle x, y-5, z-4 \rangle = 0$$

scalar equation of the plane (normal form) → $\boxed{x + 2(y-5) + 3(z-4) = 0}$

$$x + 2y + 3z = 22$$

Key: to write an equation of a plane, you need to know its normal vector

normal vector: vector perp. to the plane (was $\bar{n}$ in the example).

All planes perpendicular to $\bar{n} = \langle a, b, c \rangle$ have equations of the form

$$\underline{a}x + \underline{b}y + \underline{c}z = d, \quad d \text{ is a constant.}$$

the components of $\bar{n}$.

(note: it agrees with our example).

The number d is determined by a point on the plane.

Example: Find equation of the plane passing through $\underset{A}{(0,1,2)}$, $\underset{B}{(0,0,3)}$, $\underset{C}{(1,1,1)}$.

Solution: Need to find normal – call it $\bar{n}$

$\bar{n}$ will be perpendicular to two vectors in our plane.

- make two vectors from our points.
- use cross product.

$$\bar{v}_1 = -\overline{AB} = \langle 0, 1, -1 \rangle$$
$$\overline{BA}$$

$$\bar{v}_2 = \overline{BC} = \langle 1, 1, -2 \rangle$$

$$\bar{n} = \bar{v}_1 \times \bar{v}_2 = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = (-1)\bar{i} - \bar{j} - \bar{k}$$

$$= \langle -1, -1, -1 \rangle.$$

Answer: $-(x-0)-(y-1)-(z-2)=0$

$x+(y-1)+(z-2)=0$

(using components of A).

or:

$x+y+z-3=0$

$\boxed{x+y+z=3}$

is the same!

if not, $\vec{n}$ is wrong.

Note: also gives us the area of $\triangle ABC$: $\frac{1}{2}|\overline{AB}\times\overline{BC}|=\frac{\sqrt{3}}{2}$.

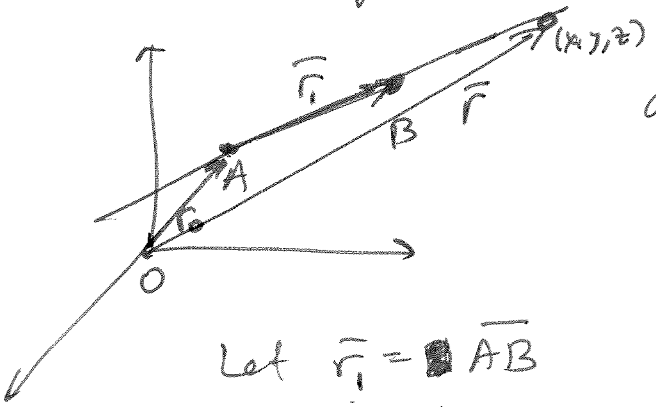
What about lines:

• two approaches: 1) equation in terms of (x,y,z) (like for the plane)

not good for plotting! $\longrightarrow$ (easy to check if a given point satisfies it).

2) parametric equation: have parameter t for any value of t get a point on the line. (good for plotting it!)

• Parametric equation of a line through A, B



at $t=0$ want to be at A

at $t=1$ want to be at B.

$\bar{r} = \langle x, y, z\rangle$ - vector from O to point (x,y,z) on the line.

Let $\bar{r}_1 = \overline{AB}$

Then $\boxed{\bar{r} = \bar{r}_0 + t\cdot \bar{r}_1}$ - vector parametric equation of the line.

Parametric equation in terms of coordinates

$$\langle x, y, z\rangle = \langle a_0, b_0, c_0\rangle + t\langle a_1 - a_0,\ b_1 - b_0,\ c_1 - c_0\rangle$$

$A = (a_0, b_0, c_0)$

$B = (a_1, b_1, c_1)$

$$\boxed{\begin{cases} x = a_0 + t(a_1 - a_0) \\ y = b_0 + t(b_1 - b_0) \\ z = c_0 + t(c_1 - c_0) \end{cases}}$$

Next: symmetric equation

Read: 10.3 - done

10.4 - almost done

10.5 - home reading.