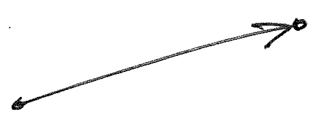
Lecture 3

Off. hrs Mon 1-2
Wed 9:30 - 10:30
+ appointments

Vectors

- What is a vector?



- magnitude + direction

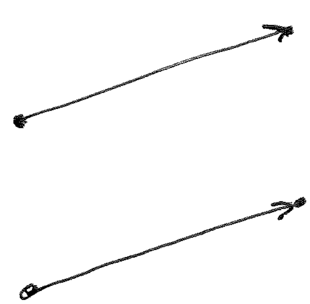
≥ 0 number

??

Equivalence class of arrows

identify (say are the same)

any two segments of the same magnitude and same direction ← works on $\mathbb{R}^2$ in $\mathbb{R}^3$

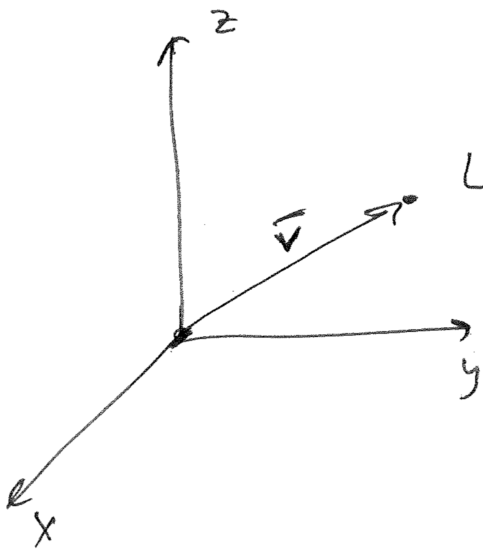


← these are the same vector

· want to describe it algebraically

Recall we have coordinate system in space.

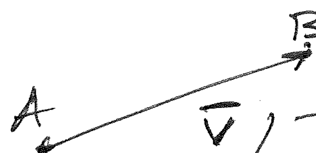
Any vector can be placed so that its starting point is at $(0,0,0)$



Let (a,b,c) be the coordinates of the end point.

Then $\bar{v} = \langle a,b,c \rangle$
$\vec{v}$

" $\bar{v}$ has components a,b,c "



$\bar{v}$ — vector notation

$\overline{AB}$

$\overline{BA}$ ← opposite to $\overline{AB}$

Another way to think of a vector in $\mathbb{R}^3$ is: just a collection of 3 numbers

$\langle a,b,c \rangle$.

Note: Points: $P = (a,b,c)$

vectors $\bar{v} = \langle a,b,c \rangle$

There is one-to-one correspondence

between points and vectors:

$$P \longleftrightarrow \overline{OP}$$

$\uparrow$ the origin

The difference is in what we can do with vectors and points.

Points just are there. Cannot add two points.

The thing we do with points is

- ask questions: does A belong to a given plane? etc.

- form vector $\overline{AB}$

With vectors, will have a lot of operations.

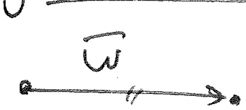
- Addition

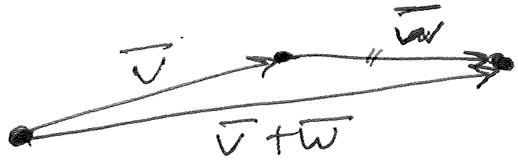
(1) $\bar{v} = \langle a_1, b_1, c_1 \rangle$

$\bar{w} = \langle a_2, b_2, c_2 \rangle$

Can define $\bar{v} + \bar{w} = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$

(2) geometrically:



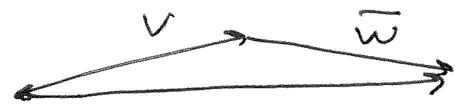


Have to prove that these definitions are the same.

Please read the proof in 10.2.

(better: come up with it!).

To prove:



compute the components of this vector.

- **Multiplication by a scalar**

$t \in \mathbb{R}$ – scalar

$\bar{v}$ – vector

$t \cdot \bar{v}$ = another vector, same direction as $\bar{v}$, magnitude equals $|t| \cdot \underbrace{|\bar{v}|}_{}$

magnitude of $\bar{v}$

In terms of components: (to prove)

$\bar{v} = \langle a, b, c \rangle$

$t \cdot \bar{v} = \langle ta, tb, tc \rangle$

• Some basic notes:

• $\bar{0}$ = the zero vector $= \langle 0, 0, 0 \dots, 0 \rangle$

• vectors in n dimensions:

$\bar{v} = \langle a_1, \dots, a_n \rangle$ – a collection (ordered) of n numbers.

same operations:

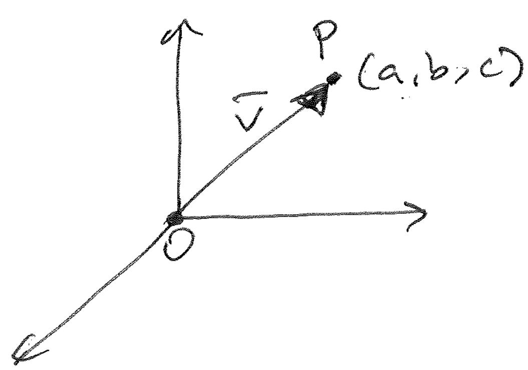
$$\langle a_1, \dots, a_n \rangle + \langle b_1, \dots b_n \rangle \stackrel{def}{=} \langle a_1 + b_1, \dots, a_n + b_n \rangle.$$

same for multiplication by scalar.

• Magnitude: $\bar{v} = \langle a, b, c \rangle$

$$|\bar{v}| = \sqrt{a^2 + b^2 + c^2}$$

Proof:



$\bar{v} = \langle a, b, c \rangle$
$\parallel$
$\overline{OP}$ where $P = (a, b, c)$

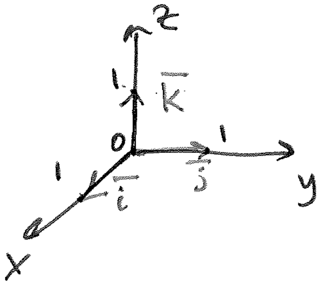
$|\bar{v}|$ = distance between O and P

$= \sqrt{a^2 + b^2 + c^2}$

by last class.

• $\bar{v}$ is called a unit vector if $|\bar{v}| = 1$.

• unit vectors of the axes: $\bar{i}, \bar{j}, \bar{k}$



$$\bar{i} = \langle 1, 0, 0 \rangle$$
$$\bar{j} = \langle 0, 1, 0 \rangle$$
$$\bar{k} = \langle 0, 0, 1 \rangle$$

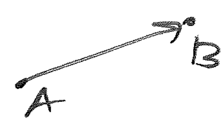
standard basis

- Writing vectors as linear combination of ~~the~~ the standard basis:

$$\boxed{\bar{v} = \langle a, b, c \rangle = a\,\bar{i} + b\,\bar{j} + c\,\bar{k}}$$

(by definitions.)

- Summary: 3 ways to think of a vector: in $\mathbb{R}^3$
  - collection of 3 components (numbers)
  - magnitude + direction



  - a combination of the basis vectors $\bar{i}, \bar{j}, \bar{k}$.

- Vector in n dimensions:

ex vector of all final exam marks (in this class, it will live in $\mathbb{R}^{66}$).

(excel works like this)

Your mark:

$$\boxed{\bar{F} = 0.15\,\bar{h} + 0.175(\bar{m}_1 + \bar{m}_2) + 0.5\,\bar{f}}$$

final mark

$\bar{m}_1$ = midterm 1 marks

$\bar{m}_2$ = midterm 2 marks

$\bar{f}$ = final exam ..

$\bar{h}$ = homework

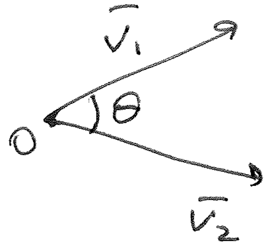
Dot product

in $\mathbb{R}^n$: $\langle a_1, \dots, a_n \rangle = \bar{v}_1$

$\langle b_1, \dots, b_n \rangle = \bar{v}_2$

Def: $\bar{v}_1 \cdot \bar{v}_2 = \underbrace{a_1 b_1 + a_2 b_2 + \dots + a_n b_n}_{\text{a \underline{number}}}$

Geometrically



set them to start at the same point
Then there is a plane containing them.

$$\boxed{\bar{v}_1 \cdot \bar{v}_2 = |\bar{v}_1| \cdot |\bar{v}_2| \cdot \cos\theta}$$

– only need to prove this in $\mathbb{R}^2$.

proving this relies on the law of cosines! (see 10.2)

Projections

- Unit vector in a given direction

$\bar{v}$ – vector

Task: make $\bar{u}$ – another vector, same direction as $\bar{v}$, but unit.

Answer:

vector.

$$\bar{u} = \frac{1}{|\bar{v}|} \cdot \bar{v}$$

number (scalar)

not dot product this is multiplication by scalar.