Fine print on Lagrange mult.

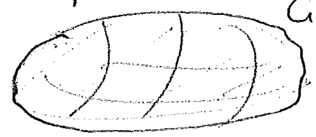
$f(x,y,z)$ - the function whose extrema we are looking for

$G(x,y,z) = 0$ - the constraint.

- For the method to work: need to assume
  - $f(x,y,z)$ is cont. differentiable
  - $G(x,y,z)$ cont. diff.
  - $\vec{\nabla} f = \lambda \vec{\nabla} G$ - really works only when $\vec{\nabla} G \neq 0$.

So, when looking for ~~others~~ candidate points for the extremum, make a list:

$G(x,y,z)=0$.



- critical points of f inside the domain ← if you have a solid in $\mathbb{R}^3$ or domain in $\mathbb{R}^2$
- On the boundary:

(1) • use Lagrange mult.

$$F = f - \lambda G$$

critical points of F

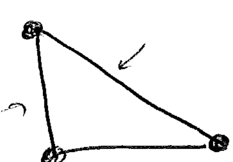
(2) All points where $\vec{\nabla} f$ does not exist (inside the domain or on the boundary)

(3) All points where $\vec{\nabla} G$ does not exist

(4) All points where $\vec{\nabla} G = 0$.

- If the boundary has "end points", these end points.

ex: $\mathbb{R}^2$, $D =$



3 different constraint functions

or

4 different Lagrange problems

Example: Find $\max f(x,y,z)$ on a cube

$$0 \le x \le 1$$
$$0 \le y \le 1$$
$$0 \le z \le 1$$

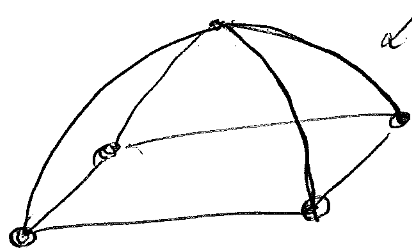
Algorithm:

- find critical pts of f inside (or on the boundary) of the cube.
- Find the candidate points on each face of the cube.

  (2 methods: Lagrange or plug in, say, $x=1$ if on the face $x=1$, work with a function of y,z).

- Look for max on the edges (plug in $x=1$, $y=0$, for example --)

  (or for more complicated solids, use Lagrange with 2 constraints)



$$f(x,y,z) - \lambda G(x,y,z) - \mu H(x,y,z)$$

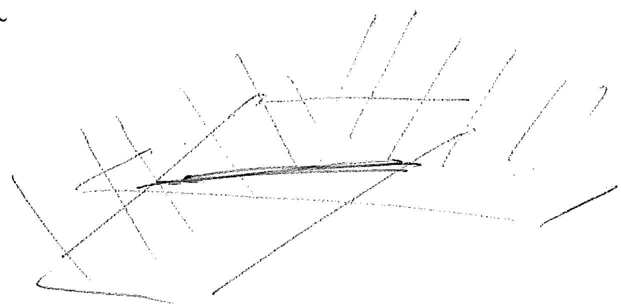
equations of surfaces whose intersection is our edge.

- Vertices
- points where f is not diff.
- points where $\bar{\nabla} G, \bar{\nabla} H, -$ are 0.

- Linear programming : shortens this process in the situation when $f(x_1 - - x_n)$ is linear and the solid ~~the~~ is defined by linear inequalities on $(x_1 - x_n)$:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \cdots \geq 0 \\ a_{2}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \geq 0 \\ \cdots \\ a_m x_1 + \cdots \qquad + a_{mn}x_n \geq 0 \end{cases}$$

(convex polyhedron).



in fact, only need to check the vertices.

(Optional reading.)

When more than one constraint:

the condition for an extremal point of $f(x_1 - x_n)$ subject to constraints $G_1(x_1 - x_n) = 0$

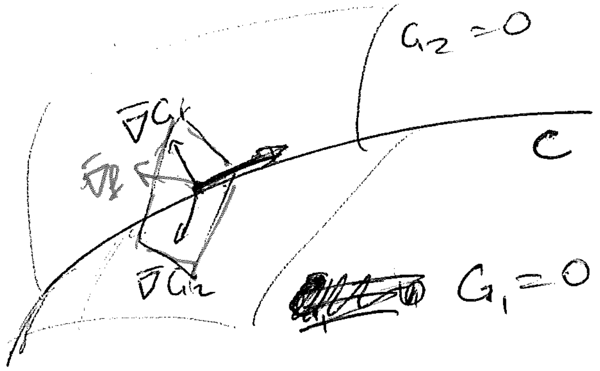
$\vdots$

$G_m(x_1 - x_n) = 0$

is: $\bar{\nabla} f$ has to lie in the subspace of $\mathbb{R}^n$ spanned by $\bar{\nabla} G_1, \ldots, \bar{\nabla} G_m$ at that point.

$(\bar{\nabla} f = \lambda_1 \bar{\nabla} G_1 + \ldots + \lambda_m \bar{\nabla} G_m)$

Lagrange multipliers



span$(\bar\nabla G_1, \bar\nabla G_2)$ = plane perpendicular to the tangent vector to the curve of intersection.

want to min/max of $f(x,y,z)$ when $(x,y,z) \in C$.

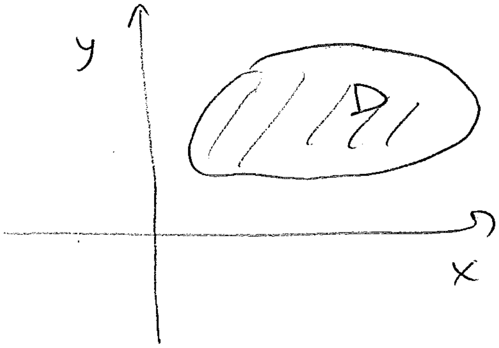
$$\bar\nabla f = \lambda \bar\nabla G_1 + \mu \bar\nabla G_2$$

Meaning of λ: it tells you how much max f will change if you replace

$G(x,y,z) = 0$ with $G(x,y,z) = h$ for small h.

max f will change by $\lambda \cdot |\bar\nabla G|$
(see p. 773 - 774)

Integrals over rectangles in $\mathbb{R}^2$ Read 14.1–14.3



want to define: $\iint_D f(x,y)\,dA$ — all one symbol.

double integral

(number of integrals = dimension of the domain)

Note: there will be no 'indefinite integral'.

(it is a single symbol).

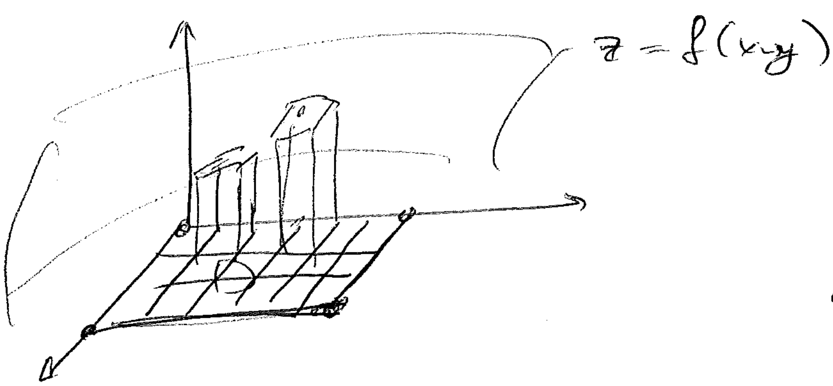
(!)

$$\left.\begin{array}{l} \iint f(x,y)\,dA \\ \iint_D f(x,y) \\ \int_D f(x,y)\,dA \end{array}\right\}$$ don't make sense.

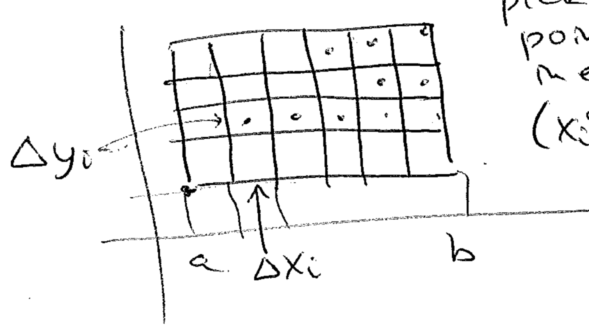
The correct usage:

$$\iint_D f(x,y)\,dA.$$

1. Let D be a rectangle.



Riemann sum: subdivide our rectangle D



pick a point in each: (x_i^*, y_j^*)

Riemann sum for the double integral is

$$\underbrace{\sum_{\substack{i=1 \ldots n \\ j=1 \ldots m}} f(x_i^*, y_j^*) \underbrace{\Delta x_i \Delta y_j}_{\substack{\text{area} \\ \text{of the small} \\ \text{rectangle}}}}_{\substack{\text{approximates the volume} \\ \text{under the graph of } f(x,y) \\ \text{over our rectangle } D.}} = R_P$$

partition

assume $f(x,y) > 0$.

"**Theorem:**" if $\Delta x_i^2 + \Delta y_j^2 \longrightarrow 0$

then all subdivisions will give Riemann sums that are very close to the volume under the graph.

($\forall \varepsilon$, $\exists \delta$ s.t. if for every i, j

diameter of each small rectangle $< \delta$

then $|R_P - R_{P'}| < \varepsilon$ for any P, P' satisfying this

and any choice (x_i^*, y_j^*).)

Then their limit exists, and is called $\iint_D f(x,y)\, dA$.

[Assume f is cont. on D])