Lecture 2

Last time: • Fixed coordinates in $\mathbb{R}^3$

• Defined $\mathbb{R}^n$.

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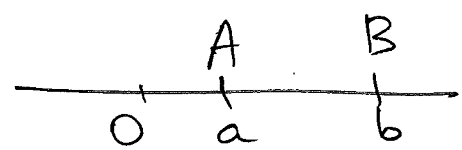
1) Distances.

Mathematically, we are free to define "distance" "any way we like"

- will do it so it agrees with the familiar notion (Euclidean distance)

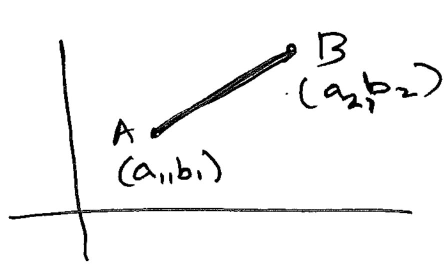
• Try to get a formula for distance between $A = (a_1, b_1, c_1)$ and $B = (a_2, b_2, c_2)$ in terms of (a_1, b_1, c_1) and (a_2, b_2, c_2).

Recall: in $\mathbb{R}^1$



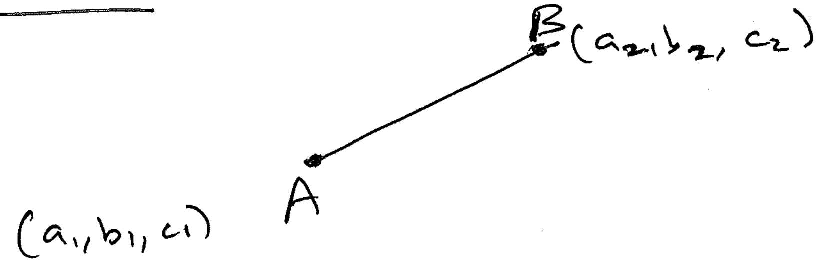
distance $|AB| = |b-a|$

in $\mathbb{R}^2$



$|AB| = \sqrt{(b_2-b_1)^2 + (a_2-a_1)^2}$

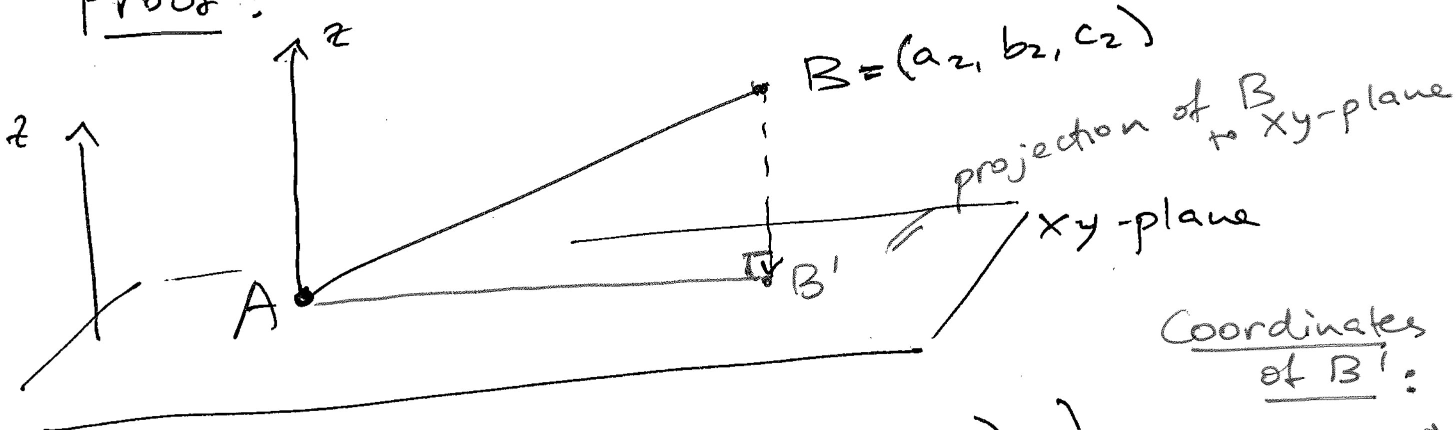
in $\mathbb{R}^3$



'<u>Conjecture</u>': $|AB| = \sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(c_2-c_1)^2}$

<u>Have to prove it</u>: has to agree with the notions of distance on every plane.

<u>Proof</u>:



<u>Coordinates of B'</u>: $(a_2, b_2, 0)$

(<u>Can assume</u> $A = (0,0,0)$.)

$$|AB| = \sqrt{|AB'|^2 + |B'B|^2}$$

$$|AB'|^2 = \left(\sqrt{(b_2-b_1)^2+(a_2-a_1)^2}\right)^2 = (a_1-a_2)^2+(b_1-b_2)^2$$

using $\mathbb{R}^2$-distance formula

$$|B'B|^2 = |c_2|$$

$$|AB| = \sqrt{(a_2-a_1)^2+(b_2-b_1)^2 + \underset{\overset{\shortparallel}{(c_2-c_1)^2}}{c_2^2}}$$

↑ assuming A was in the xy-plane so $\cancel{a}\ c_1 = 0$

Distance in $\mathbb{R}^n$

$A = (a_1, a_2, \ldots, a_n)$ $\qquad$ $B = (b_1, b_2, \ldots, b_n)$

$$|AB| = \sqrt{(a_1-b_1)^2 + \ldots + (a_n-b_n)^2}$$

Equations and inequalities in $\mathbb{R}^3$

1) Equation of a sphere of radius R centered at $A=(a,b,c)$:

<u>Recall</u>: sphere = set of points in $\mathbb{R}^3$ whose distance from $A=(a,b,c)$ equals R

$$= \{P=(x,y,z) \mid |AP| = R\}$$

$$= \{(x,y,z) \mid \underbrace{(x-a)^2+(y-b)^2+(z-c)^2 = R^2}_{\text{equation of the sphere!}}\}.$$

2) Planes in $\mathbb{R}^3$:

$(x, y, z) \in \mathbb{R}^3$

$\boxed{ax + by + cz = d}$ - general form of equation of a plane

(by analogy with line in $\mathbb{R}^2$:
$ax + by = c$)

(!) though it is analogous to a line in $\mathbb{R}^2$, it defines a plane not a line in $\mathbb{R}^3$!

- reasonable that one equation in 3 variables will define a 2-dimensional object.

3) How to define a line?

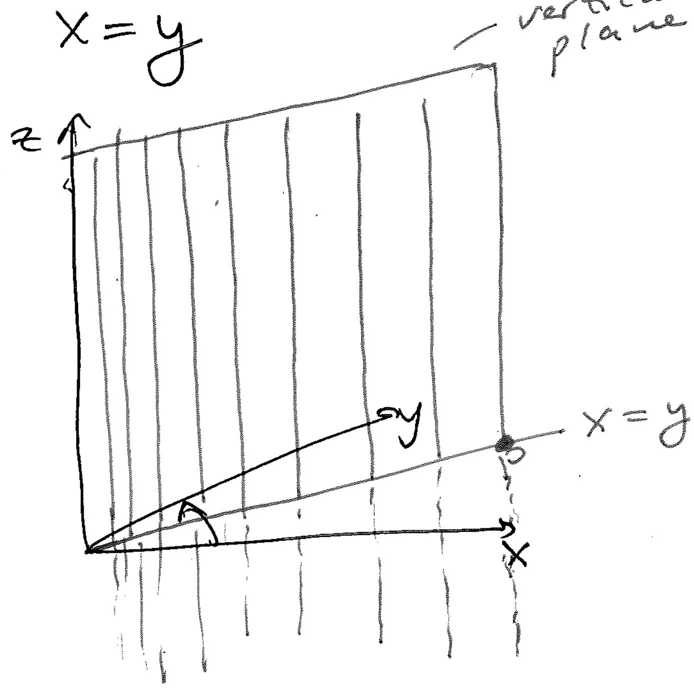
Need 2 equations, e.g.

$$\begin{cases} a_1 x + b_1 y + c_1 z = d_1 \\ a_2 x + b_2 y + c_2 z = d_2 \end{cases}$$

- intersection of two planes = line. (usually).

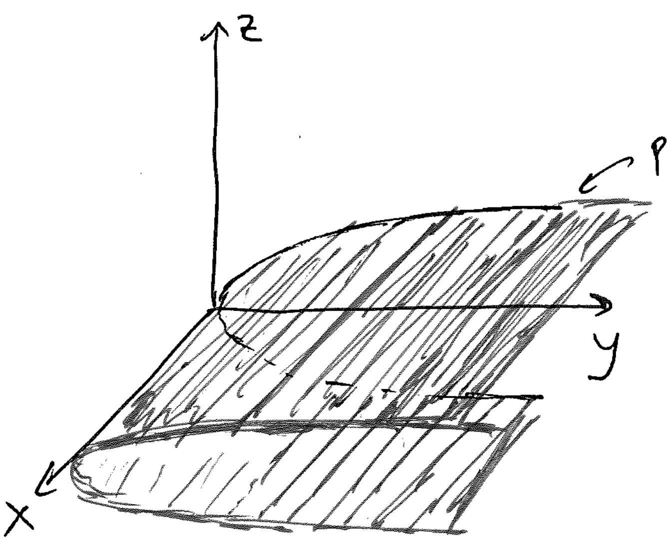
4) Examples of equations: (when one variable is missing)

(a) $x = y$



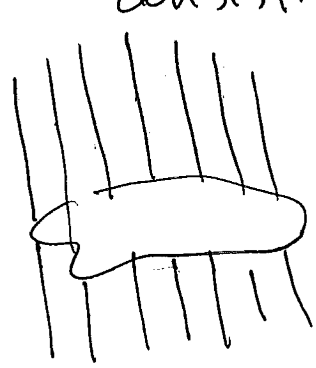
Since z does not participate in the equation, for any point $P = (x, y, 0)$ there is a whole vertical line through P

(b) $z^2 = y$



parabolic cylinder

Cylinder: any surface consisting of parallel lines.

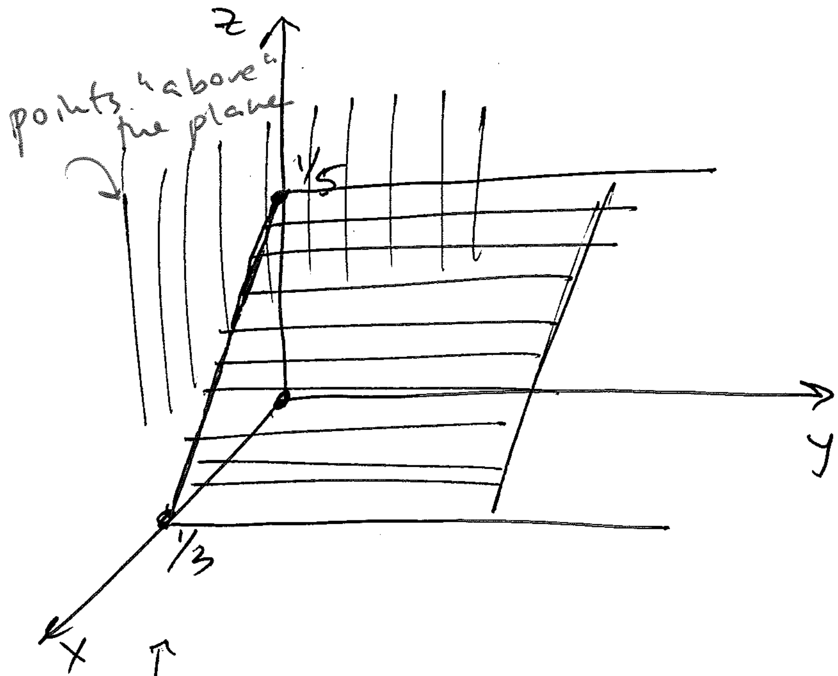


– plane curve + line gives a cylinder.

Inequalities:

Ex: $3x + 5z \geq 1$ – half-space

$3x + 5z = 1$ = plane parallel to y-axis



first octant