* Use ε-δ once to prove cont. of basic functions (of 1 variable) — go back to our book and find proofs.

(ex: revisit $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ - prove using ε-δ)

* product, sum, quotient (if denom. $\neq 0$) — review
* Will now talk about

Suppose I have $f(x)$ and $g(y)$ - continuous single variable functions.

$h(x,y)$ - cont. fn of 2 variables.

Make $h(f(x), g(y)) = F(x,y)$. Then it is cont.

EX: $h(x,y) = x\cdot y$ ← rename $h(u,v) = u\cdot v$

Now take $f(x) = e^x$ $g(y) = \sin(y)$

$h(f(x), g(y)) = e^x \sin y$.

Then can conclude $h(f(x), g(y))$ is continuous.

Proof: Want to prove: f, g, h are continuous then $F(x,y) = h(f(x), g(y))$ is also continuous.

Discussion of domain: (x_0, y_0) in the domain of F, mean:
- x_0 is in the domain of f
- y_0 is in the domain of g

and $(f(x_0), g(y_0))$ is in the domain of h.

Proof: Take (x_0, y_0) in the domain of $F(x,y)$. Given $\varepsilon > 0$, want to prove that exists $\delta > 0$ such that when $(x,y) \in \underbrace{B_\delta(x_0,y_0)}_{\text{disc of radius } \delta \text{ centred at } (x_0,y_0)}$ then $|F(x,y) - F(x_0,y_0)| < \varepsilon$.

Our task: Given ε, find such δ.

Detective work: unwind the definition of $F(x,y)$.

$$F(x,y) = h(f(x), g(y)).$$

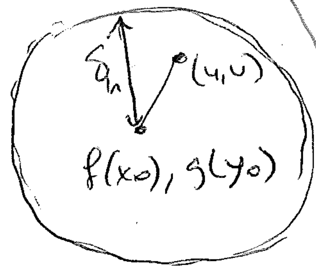
Since h is continuous, we know that

$\exists\, \delta_h$, s.t. if $|f(x) - f(x_0)| < \delta_h$

and $|g(y) - g(y_0)| < \delta_h$,

then

$$|h(f(x_0), g(y)) - h(f(x_0), g(y_0))| < \underline{\varepsilon}.$$



$|h(u,v) -$

$- h(f(x_0), g(y_0))| < \varepsilon.$

Now we need to find a δ, such that when $|x - x_0| < \delta$ and $|y - y_0| < \delta$, then the point $(u,v) = (f(x), g(y))$ lies in the circle of radius δ_h centered at $(f(x_0), g(y_0))$.

Now use that $f(x)$, $g(y)$ are continuous:

find δ_f and δ_g s.t. when

$|x - x_0| < \delta_f$, then $|f(x) - f(x_0)| < \frac{\delta_h}{\sqrt{2}}$,

and

when $|y-y_0| < \delta_g$, then $|g(y) - g(y_0)| < \frac{\delta_h}{\sqrt{2}}$

~~Finally,~~ Finally, when $|x-x_0| < \delta_f$
$|y-y_0| < \delta_g$,

then: distance from

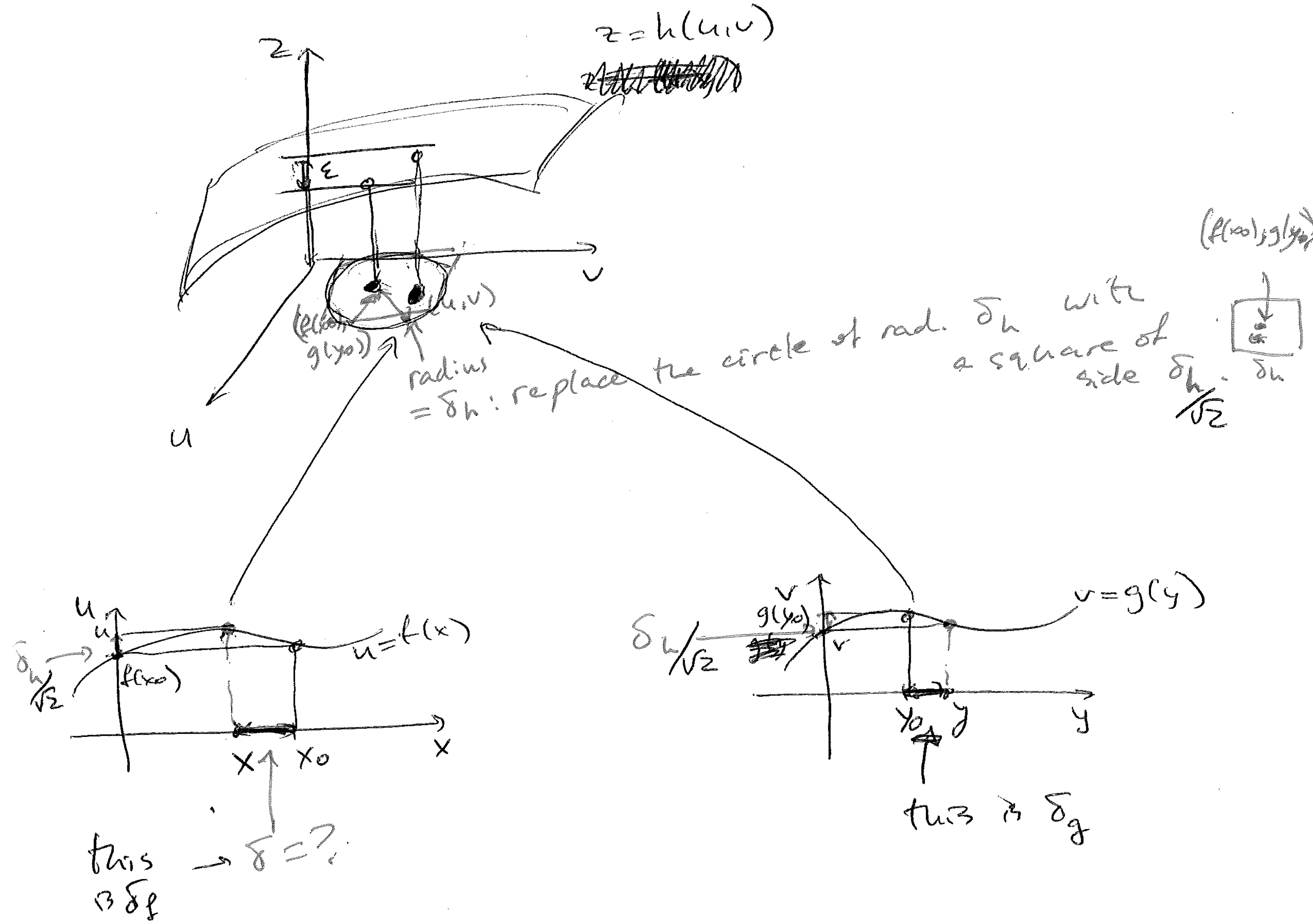
$(f(x), g(y))$ to $(f(x_0), g(y_0))$

is less than δ_h.

And then $|h(f(x), g(y)) - h(f(x_0), g(y_0))|$
$< \varepsilon$.

Then we choose $\delta = \underline{\min(\delta_f, \delta_g)}$

$\boxed{\text{See picture on the next page}}$


z = h(u,v)
z
ε
v
u
(f(x0), g(y0))
(u,v)
radius = δh: replace the circle of rad. δh with a square of side δh/√2
(f(x0), g(y0))
δh
u
u = f(x)
f(x0)
δh/√2
x
X
x0
this is δf
δ = ?
v
v = g(y)
g(y0)
δh/√2
y0
y
this is δg

Partial derivatives (12.3)

do not read the part about "chain rule".

$f(x,y)$

Define f_1 (or f_x) or $\frac{\partial f}{\partial x}$ – derivative of f with respect to x

and f_2 (or f_y) or $\frac{\partial f}{\partial y}$ – / — w.r.t. y

– to actually do it: keep y fixed, think of x as a variable and differentiate. This ~~...~~ gives f_x.

Fix x, differentiate (as a fn of y) – get f_y.

Example: $f(x,y) = x^2 \sin y$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 \sin y) = 2x \cdot \sin y$$

notation

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 \sin y) = x^2 \cdot \cos y$$

x treated as const!