**Department of Mathematics**

**University of British Columbia**

**MATH 226 Final Exam**

**December 4, 2013, 12:00PM - 2:30PM**

Family Name: ______________________________ Initials: _____

I.D. Number: _______________ Signature: ____________________________

**CALCULATORS, NOTES OR BOOKS ARE NOT PERMITTED.**

**JUSTIFY ALL OF YOUR ANSWERS (except as otherwise specified).**

**THERE ARE 8 PROBLEMS ON THIS EXAM.**

$\log(x)$ **MEANS THE NATURAL LOGARITHM OF** $x$**.**

| Question | **Mark** | **Out of** |
|---|---|---|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 10 |
| Total | | 80 |

1.

(a) Find an equation of the tangent plane to the surface $2x^2 + 3y^2 + 4z^3 = 9$ at the point $(1, 1, 1)$.

(b) Find an equation of the tangent plane to the surface $z = 2x^2 - y^3$ at the point $(1, 1, 1)$.

(c) Find an equation of the tangent line to the curve of intersection of these surfaces at the point $(1, 1, 1)$.

2.

(a) Let $A$ be an $m \times n$ matrix. Let $F : \mathbb{R}^n \to \mathbb{R}^m$ be defined by

$$F(\mathbf{x}) = A\mathbf{x}^T$$

Find the Jacobian, $DF$, of $F$, in terms of $A$.

(b) Let $B$ be a *symmetric* $n \times n$ matrix. Let $g : \mathbb{R}^n \to \mathbb{R}$ be defined by

$$g(\mathbf{x}) = \mathbf{x}B\mathbf{x}^T$$

Find the Hessian of $g$ in terms of $B$.

3.

Evaluate

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dydx$$

4.

Re-iterate the integral

$$\int_{x=1}^{2} \int_{y=x}^{2} \int_{z=\log x}^{\log y} f(x, y, z)\ dzdydx$$

in the following orders by filling in the upper and lower limits of the integrals.

(a)

$$\int \int \int f(x, y, z)\ dxdzdy$$

(b)

$$\int \int \int f(x, y, z)\ dydxdz$$

NO justification required.

5.

Evaluate $\int\int\int_D x^2 + y^2 + z^2 \, dV$ where $D$ is the solid lying inside the sphere of radius 1 centered at $(0, 0, 1)$ and inside (i.e above) the cone $x^2 + y^2 = 3z^2$.

6.

Give $\epsilon - \delta$ proofs to justify the following statements.

(a) $f(x, y) = \sqrt{x^2 + y^4}$ is continuous at $(0, 0)$.

(b) $g(x, y) = x^2 y$ is differentiable at $(1, 1)$.

For part b, you do NOT need to justify computation of partial derivatives with $\epsilon - \delta$ proofs.

7. (a) Show that the intersection of any two open sets is open.
   (b) Show that the intersection of any two closed sets is closed.
   (c) Show that the boundary of any set is closed.

8.

Find the absolute maxima and absolute minima (and their values) of the function

$$f(x, y, z) = x \log x + y \log y + z \log z$$

subject to the constraints $x + y + z = 1$, $x \geq 0, y \geq 0, z \geq 0$.

You may assume $0 \log 0 = 0$.